Dynamic Networks and Asset Pricing

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ABSTRACT

In the context of an equilibrium model with multiple risky assets, we map the characteristics of the network structure under incomplete information to the cross-section of expected returns. We show the existence of a link between the way firms are connected in a network and both P/D ratios and expected returns. Firms that are more 'central' have lower P/D ratios and higher expected returns. We take the model to the data and use information on dividends to estimate the network structure. Then, we test two predictions of the structural model. First, exogeneity is priced in the cross-section, even after controlling for the Fama an French factors. Second, network structure helps to explain cross-sectional predictability (industry momentum and reversal) as reported in the literature. The results support a structural interpretation of these two empirical regularities.

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Paolo Porchia gratefully acknowledges financial support from the Ministerio de Ciencia e Innovación de España (Spanish Ministry of Science and Innovation) under the 'Ramón y Cajal' program (RYC-2010-07401) and under grant number ECO2011-26384.

I Introduction

This paper studies the effects of the network structure implied by firms' cash flows on the cross-section of expected returns. An increasing literature in economics investigates the role of interconnections between different firms and sectors, functioning as a potential propagation mechanism of idiosyncratic shocks throughout the economy. Acemoglou et al. (2011) use network structure to show the possibility that aggregate fluctuations may originate from microeconomic shocks to firms. Such a possibility is discarded in standard macroeconomics models due to a "diversification argument". As argued by Lucas (1977), among others, such microeconomic shocks would average out and thus, would only have negligible aggregate effects. Similarly, these shocks would have little impact on asset prices. For instance an investor holding a diversified portfolio of firms in the symmetric network of Figure 1 would not be exposed to each of these firm specific shocks as the number of firms grows large.

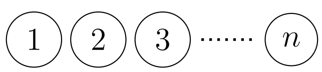


Figure 1a

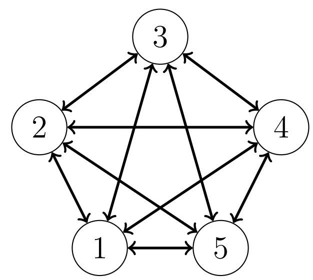


Figure 1b

Figure 1a describes a disconnected orchard; Figure 1b describes a symmetrically connected network.

A different situation emerges in the network of Figure 2a. In this case firm 1 is central in the network so that shocks to this firm propagate to the rest of the network. Even if an investor were to hold a portfolio of n stocks (for $n \to \infty$), she would still be exposed to shocks of firm 1. Similarly, in the network of Figure 2b she would be exposed to shocks of the $\overline{n}$ central firms. This becomes important in terms of the cross-sectional properties of expected returns. The centrality of a firm makes observations of shocks to its dividend important for expected returns on other stocks.

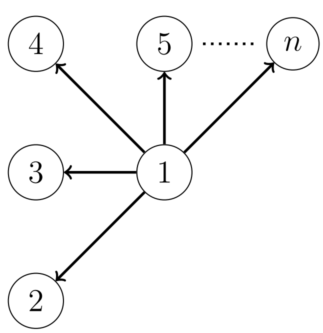


Figure 2a

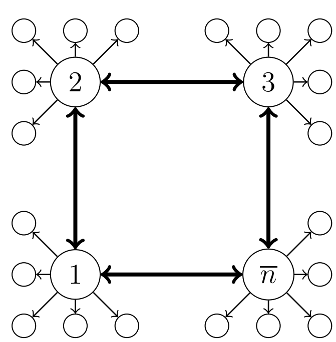


Figure 2b

Figure 2a describes a "Star" network with firm 1 being the central node. Figure 2b describes a clustered network with n connected "Stars".

The idea of the existence of an asymmetric network structure has been at the heart of much of policy decisions in the U.S. and Europe during the 2008 Credit Crisis, when several financial firms have been rescued to avoid adverse effects to the rest of the economy. Substantial regulatory effort is now been devoted to understand linkages across firms and sectors to limit excessive propagation of shocks. Mervyn King, the governor of the Bank of England, called for banks that are "too big to fail" to be cut down to size, as a solution to the problem of banks having taxpayer-funded guarantees for their speculative investment banking activities. "If some banks are thought to be too big to fail, then, in the words of a distinguished American economist, they are too big. It is not sensible to allow large banks to combine high street retail banking with risky investment banking or funding strategies, and then provide an implicit state guarantee against failure." Alan Greenspan echoed to these statement saying: "I don't think merely raising the fees or capital on large institutions or taxing them is enough ... they'll absorb that, they'll work with that, and it's totally inefficient and they'll still be using the savings."

In this paper, we study a Lucas economy with multiple trees producing a dividend that is subject to both idiosyncratic shocks, which follow a tree-specific Markov chain, and a common business cycle shock. We depart from the literature by allowing for the interaction of two channels: (a) incomplete information with learning; and (b) an asymmetric cash-flow connectivity structure. Agents do not observe whether the shocks are idiosyncratic or systematic as they only observe firms' dividend. From these observations they infer the state of the economy. This implies that shocks to one firm affect expected returns on all other firms. Incomplete information and cross-sectional learning becomes a source of comovement in asset prices in the network even if shocks have hit a single firm only. The second channel introduces network connectivity by allowing each firm to have heterogeneous and state-dependent intensity of distress (the parameters of the Markov chain). This allows to have a network structure in which firms propagate and/or absorb shocks differently. Once this is coupled with incomplete information and cross-sectional learning, the model gives rise to interesting implications in terms of the link between firms' connectivity and the cross-section of expected returns. In our model, we show that the equilibrium risk premium of a firm becomes positively related to the extent that shocks to its dividend affect the posterior beliefs of other firms' dividend (and to the extent that the opposite is not true). These securities are *actively* connected to the rest of the network, in that they can transfer their own shocks while being relatively insulated from other firms' shocks. In equilibrium, the relative hedging demand of actively and passively connected stocks is different, giving rise to a greater risk premium of actively connected stocks (such as firm 1 in Figure 2a), with respect to passively connected stocks (such as firm 2 in Figure 2a). We propose a reduced-form univariate measure of firm connectivity that capture information that is relevant for expected returns, which we call "dynamic centrality". We show that the larger the dynamic centrality the larger the expected returns and the smaller the P/D ratio, thus suggesting a structural link between network structure and the cross-section of expected returns.

We use data on dividends to estimate the characteristics of the network structure by maximum likelihood. Since dividends are distributed infrequently we aggregate firms in portfolios by mimicking Fama and French (1992) methodology and find that that after controlling for market beta, size and book-to-market, centrality has a positive and significant price of risk. An increase of one standard deviation of the measure implies an approximate 0.18% decrease of the monthly risk premium. A portfolio that is short stocks in the first quintile and long stocks in the last quintile of the empirical distribution of centrality gains an average monthly return of 1.7%. A time series regression shows that this return cannot be fully attributed to the Fama-French (1992) factors, as a significant alpha of 1.26% emerges. At the same time, we find that centrality and network connectivity helps to explain a significant part of the cross-section of the value premium: more exogenous firms have larger book-to-market. This is very interesting since it offers a structural explanation to a well known empirical regularity.

The second implication of the model is related to cross-sectional momentum. Menzly and Ozbas (2011) find that returns of an industry are positively related to past returns of industries connected by a supplier or customer relationship. They use data from input-output BEA tables and argue that predictability emerges for these firms when analyst coverage is scarce and past connected returns help resolve uncertainty. We use our model-implied notion of cash-flow connectivity to distinguish between incoming connections and outgoing connections. We find that after controlling for the strength of the incoming connectivity, i.e. to which extent the fundamentals of the firm are caused by its most connected firms, a portfolio long (short) stocks whose connected performers did best (worst) gains abnormal returns. Our network environment is consistent with cross-momentum for the most exogenous stocks and cross-reversal for the least exogenous. This is interesting since it suggests that network connectivity helps to explain an important well-known regularity in the data.

Related Literature. This paper is related to three streams of the literature. A first stream studies the implications of multiple trees (orchard) in asset prices. Cochrane, Longstaff, and Santa-Clara (2008) show that even if dividends are lognormal i.i.d., simple market clearing can give rise to complex asset price dynamics. Martin (2011) extend the results to a general set of processes. Our model explores, instead, the implications of network connectivity. This is motivated by the fact that some empirical properties in the data are difficult to be reconciled with traditional models. Santos and Veronesi (2009), for instance, show that the SDF implied by nonlinear external habit formation preferences counterfactually generates higher expected returns for stocks with high price-dividend ratios – i.e. a 'growth premium' – if firms/trees are allowed to differ only in terms of their expected dividend growth. They show that the 'value premium' can be obtained as long as one introduces heterogeneity in the firms' cash-flow risk, that is, in the covariance between consumption growth and their dividend growth. A related result is obtained in Lettau and Wachter (2007), who advocate the importance of weak or positive covariance between the market price of risk and dividend shocks, in order to obtain a 'value premium'. Our contribution is to show both theoreticaly and empirically a structural channel that is independent of preferences that is consistent with some empirical regularities.

A second stream of the literature studies the role of sectorial shocks in macro fluctuations; examples include Horvath (1998, 2000), Dupor (1999), Shea (2002), and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2011). This literature centers around whether sectorial shocks would translate into aggregate fluctuations. While these studies present important empirical evidence on the role of sectorial shocks, very little is known about their asset pricing implications in asymmetric networks. In order to address this question, we introduce dynamics in a simple network by using connected markov chains. If previous studies focus on shock propagation in static networks, to study asset prices we also allow agents to have incomplete information on the sources of the shocks (whether being idiosyncratic or systematic) and investigate the propagation of shocks in the cross-section. The network connectivity that we study is related to the role played by firm size distribution in Gabaix (2011) who shows that firm-level idiosyncratic shocks translate into aggregate fluctuations when the largest firms contribute disproportionally to aggregate output. While this could indeed be the case in our network structure, the specification of our model is in terms of input-output linkages. We study the role of firm size in the empirical section.

A third stream of the finance literature studies the role of idiosyncratic risk in asset pricing and cross-sectional momentum. The CAPM predicts that only systematic risk is priced and expected excess stock returns satisfy a two fund separation property. While this is indeed the case in the context of a symmetric network, as in Figure 1a and 1b, this is no longer true in a network in which some firms have a central position, as in Figure 2. This feature is related to the findings in Ang, Hodrick, Xing, and Zhang (2006) who find that volatility risk is priced in the cross-section of expected stock returns, a regularity which is not subsumed by the standard size, book-to-market, momentum effects, or liquidity effects. Moreover, Menzly and Ozbas (2011) find that returns of an industry are positively related to past returns of industries connected by a supplier or customer relationship. Our model produces implications that are consistent with their results.

The article is organized as follows. Section II describes the model and the agent's learning mechanism. Section III derives security prices and equity premia, relating their cross-sectional behavior to network connectivity and incomplete information. Section IV discusses the link between network centrality and the fund-separation property of expected returns. Section V describes the 'centrality' measure in charge of testing empirically network connectivity. Sections VI and VII contain the empirical analysis: the former is concerned with 'centrality' and the cross-section of returns, the latter with cross-momentum. Section VIII concludes. Proofs and additional expressions not reported in the text are in Appendix A. Appendix B reports details of the estimation procedure of Sections VI and VII, and of a calibration discussed in Section V.

II The Economy

In an infinite-horizon, pure exchange Lucas economy a representative agent maximizes Constant Relative Risk Aversion utility of intertemporal consumption.

$$U_0 = \mathbb{E}\left[\int_0^\infty e^{-\delta s}\frac{C_s^{1-\gamma}}{1-\gamma}ds\right]. \tag{1}$$

γ and δ are the relative risk aversion coefficient and the subjective discount rate, respectively. The opportunity set of the investor consists of a locally risk-less security in zero net supply, with rate of return r_t (the interest rate), and n risky securities in positive net supply, each paying a stochastic dividend stream D_t^i, $i = 1, \ldots, N$. Prices satisfy the market clearing condition that aggregate consumption C_t is equal to the sum of the dividend processes D_t^i plus an endowment flow (labour income) L_t: $C_t = \sum_{i=1}^n D_t^i + L_t$. We will treat the endoment flow as the $n+1-$th dividend process, although it is not the claim of a risky security. Single trees in this Lucas orchard can be, e.g., sectors or individual firms in a domestic economy, or countries in an international framework. Each endowment is composed of a smooth, lognormal dividend factor Y_t^i and of jump component x_t^i, independent of Y_t^i:

$$\begin{aligned} D_t^i &= Y_t^i x_t^i, \qquad i = 1, \ldots N+1, \\ \frac{dY_t^i}{Y_t^i} &= \mu^i dt + \sigma^i dZ_t, \end{aligned} \tag{2}$$

While Y_t^i is continuous, x_t^i follows a simple jump process, where positive (recovery) jumps can only follow negative (distress) ones, and they all have constant logarithmic size J^i.[1] Thus jump events are actually transitions to and out of a persistent state of distress, captured by the binary variable H_t^i, which takes value 1 if tree i is in distress state, and 0 if it is not. The persistence of the state is governed by the distress (recovery) intensity λ_t^i (η_t^i), which is the instantaneous probability of a negative (positive) jump, provided the tree is not (the tree is) in distress. The long-run persistence of the states is important in our context, because we are interested in the cross-sectional propagation and asset pricing implications of localized distress events in a connected network. Expression (2) implies that dividends evolve in time as:

$$\frac{dD_i}{D_i} = \mu^i dt + \sigma^i dB_i + (e^{-J^i} - 1)(1 - H_{t-}^i)dH_t^i - (e^{J^i} - 1)H_{t-}^i dH_t^i \tag{3}$$

An innovation $dH_t^i = 1$ denotes a distress jump of tree i, while $dH_t^i = -1$ denotes a recovery jump. If H_t was independent across trees, the economy would be described by Figure 1 and the network would be a traditional disconnected orchard.[2]

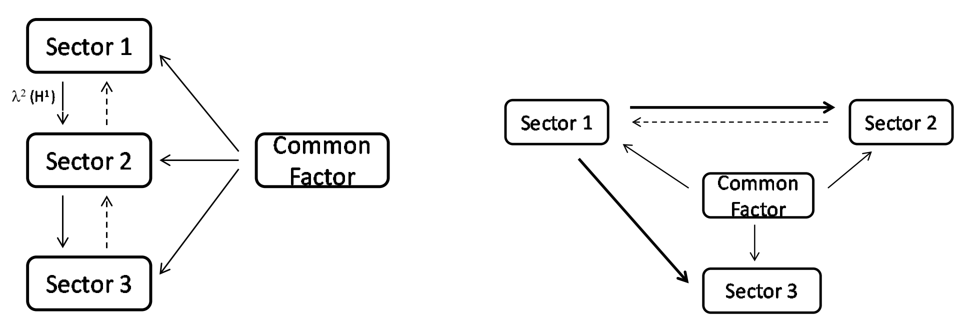
[1] This homogeneity assumption is not essential and it will be later generalized in the empirical section.

[2] Formally, the process x_t^i, is given by $x_t^i = x_0^i \prod^{nd(0,t_-)} e^{-J^i} \prod^{nr(0,t_-)} e^{J^i}$, with $nd(0, t_-)$ and $nr(0, t_-)$ being the number of past distress and recovery jumps, respectively.

We now introduce connectivity in the network. We assume that the intensities depend on a common latent business cycle factor S, and on the state of distress of the other trees: $\lambda^i(S, \mathbf{H})$, $\eta^i(S, \mathbf{H})$.[3] The trees are then part of a (production) network with two layers of connectivity: *a*) a systematic connectivity, via the business cycle factor S, which bypasses individual connections by exercising common influence on all dividend growths; *b*) a local connectivity, which allows endowments to be positively or negatively affected by the state of distress of directly connected trees. The latent factor S is a Markov chain with a recession ($S = 1$) and a boom state ($S = 0$), and transition intensities k_h and k_l, respectively. When S is in a recession state, the distress events of the trees are always more likely, and recoveries are less likely, so that $\lambda_i(1, \mathbf{H}) \geq \lambda_i(0, \mathbf{H})$ and $\eta_i(1, \mathbf{H}) \leq \eta_i(0, \mathbf{H})$. We assume that S is unobservable by the agent, so that the agent is uncertain about whether a shock to a tree is truly systematic and can generate contagion: a distress may be indicative of a recession ($S = 1$), where all trees are more prone to distress and it is legitimate to expect more, or it may have happened idiosyncratically during a boom ($S = 0$). The converse would be true for a recovery event. Shocks to a tree are used by the agent to form posterior beliefs about other trees' fundamentals, because of the common influence of the business cycle factor. The updating depends on the way trees are connected. Thus, isolated shocks to a single tree can have immediate cross-sectional implications which depend on the network structure.

Examples. To provide a simple intuition of how the model specification can be used to characterize different networks, in Figure 3 we display two examples. In the first, Sector 1 is upstream in terms of cash-flow shocks, while Sector 3 is downstream. This could be used to represent a vertically integrated value chain where shocks flow more quickly downstream (solid rather than dashed arrows). In this case $\lambda^2(H_t^1 = 1) > \lambda^1(H_t^2 = 1)$ and $\lambda^3(H_t^2 = 1) > \lambda^2(H_t^3 = 1)$.

In the second network, on the right, we consider the case of a network in which Sector 1 transmits shocks to other sectors but not vice-versa. This is displayed by solid arrows that can be formalized with large values of $\lambda^j(H_t^1 = 1)$ for $j \neq 1$, and small $\lambda^1(H_t^j = 1)$.



Vertical Integration *Sector 1 Dynamic Centrality*

Figure 3. The panel on the left describes a vertically integrated network; the panel on the right describes a network in which Sector 1 is exogenous.

[3] When no confusion can arise we adopt the notation $\mathbf{H}_t = (H_t^1, \ldots, H_t^{n+1})$.

A Learning

Agents observe dividend jumps, hence realizations of $\mathbf{H}_t$, but they do not know the state of the business cycle variable S, thus they do not know the intensities λ^i and η^i. They estimate them bayesianly from past jump events observing the whole network. Let $p_t^h = P(S_t = 0|\mathcal{F}_t^{x,Y})$ be the posterior belief that the economy is not in a bad systematic state. $\mathcal{F}_t^{x,Y}$ denotes agent's available information. It follows that the agent's perceived intensities of distress and recovery jumps are, respectively:

$$\widehat{\lambda}^i(\mathbf{H}_t) = \mathbb{E}[\lambda^i(\mathbf{H}_t)|\mathcal{F}_t^{x,Y}] = p_t^h\lambda^i(0,\mathbf{H}_t) + (1-p_t^h)\lambda^i(1,\mathbf{H}_t) \tag{4}$$

$$\widehat{\eta}^i(\mathbf{H}_t) = \mathbb{E}[\eta^i(\mathbf{H}_t)|\mathcal{F}_t^{x,Y}] = p_t^h\eta^i(0,\mathbf{H}_t) + (1-p_t^h)\eta^i(1,\mathbf{H}_t) \qquad i = 1,\ldots,n+1 \tag{5}$$

The filtered dynamics of the posterior probability p_t^h for the latent factor are given in the next lemma.

Lemma 1. *Let p_0^h denote investor's prior belief that $S = 0$. The posterior probability dynamics of p_t^h follows the stochastic differential equation:*

$$dp_t^h = \left[k_l - (k_l + k_h)p_t^h\right] dt + p_t^h(1-p_t^h)\sum_{i=1}^{n+1}\Big[(\lambda^i(1,\mathbf{H}_{t-}) - \lambda^i(0,\mathbf{H}_{t-}))(1-H_{t-}^i)d\hat{H}_t^i + (\eta^i(1,\mathbf{H}_{t-}) - \eta^i(1,\mathbf{H}_{t-}))H_{t-}^i d\hat{K}_t^i\Big] \tag{6}$$

where

$$d\hat{H}_t^i := \frac{dH_t^i - \widehat{\lambda}_t^i dt}{\widehat{\lambda}_t^i}, \quad d\hat{K}_t^i := \frac{-dH_t^i - \widehat{\eta}_t^i dt}{\widehat{\eta}_t^i}, \tag{7}$$

is the process of distress and recovery innovations of tree i, $i = 1,\ldots,n+1$, such that

$$E[d\hat{H}_t^i|\mathcal{F}_t^{x,Y}] = 0 \quad \text{and} \quad E[d\hat{K}_t^i|\mathcal{F}_t^{x,Y}] = 0.$$

Expression (6) is intuitive. The stochastic components $d\hat{H}_t$ and $d\hat{K}_t$, are the normalized unexpected innovations of distress and recovery realizations. If the distress intensities were independent of the business cycle (i.e. $\lambda(1,\mathbf{H}) = \lambda(0,\mathbf{H})$) , the events would be firm-specific and idiosyncratic: signals would be uninformative and no cross-sectional learning would arise. When distress is more likely in recession, however, the observation of such an event for a tree leads to a downward revision of the posterior probability p_t^h over the cross-section. On the other hand, an upward revision takes place in case of recovery event, if the latter is less likely in recession. Note that distress and recovery signals are realized discretely over time. Therefore the posterior probability have discontinuous trajectories. Between any two updates the process is dominated by the drift component, which implies a reversion to the long-term mean $\overline{p} = k_l/(k_l + k_h)$ at speed $\theta = k_l + k_h$. $\overline{p}$ is simply the fraction of time that the economy spends in a good state in the long run. The speed of reversion to the mean is larger when transition probabilities are higher and boom-burst cycles shorter. This also implies that the impact of bad news in good times is larger than in bad times.

Since distress and recovery innovations enter equation (6) weighted by the difference of disaster and recovery intensities in good and bad states, individual distress and recoveries have greater weight in investors' posterior distribution whenever the underlying individual intensity process is more volatile across, hence correlated with, the business cycle factor. In addition, all signals have a uniformly greater weight when overall Bayesian uncertainty is large, i.e., when the term $p_t^h(1-p_t^h)$ is large. This occurs for $p_t^h \approx 0.5$, when investors face the largest degree of subjective uncertainty about the true common latent state of the economy. These features can generate interesting effects of perceived distress contagion via agents' optimal learning behavior: large revisions in the posterior intensity of a tree i can arise because of the observation of a distress or recovery in another tree $j \neq i$, even in absence of any cash flow innovations for tree i. An example in the next subsection illustrates the point.

B Supply-Chain and Industry Networks

It is interesting to study the conditions in which 'distress' of a few trees can propagate through the chain of technologically connected firms. If, for instance, some industrial sectors perform specific functions, the quality and/or magnitudes of shock transmissions depend on the specific identity of the sector which suffered the initial shock. Allowing jump intensities to be contingent on the state of distress of other trees incorporates network structures in a simple form. As an illustration, consider an economy with two sectors, Banking and Manufacturing. Suppose that Manufacturing depends on Banking through the supply-side credit channel, so that distress of the latter exposes to distress the former. The opposite is true to a lesser extent, but in recession the link becomes closer in both directions, in accordance with the well documented fact that correlations increase in periods of market downturns. Jump intensities in Panel 1 of Table VIII are consistent with this simple network, depicted in Panel 2.

Insert Table VIII (8)

Panel 3 shows the evolution in time of the perceived probability that both sectors will be in distress in 1 year.[4] The solid line reports the disconnected case, where network structure is ignored, while the dotted line is obtained taking dependencies into account. Intuitively, the direct propagation of shocks through technological connections increases the probability of contagion and distress clustering, relative to the disconnected case. Moreover, the informational contagion described in the previous Section is much more evident. The posterior update at each endowment jump is greater when sectors are believed to be more closely connected in a recession, because the same bad news induces agents to take a more pessimist stance on the other Sectors.

[4]The expression of this probability is reported in Appendix A.

III Model results

Cross-sectional learning and network structure give rise to interesting asset pricing implications for both valuation ratios and risk premia. One of these implications is a form of cross-sectional predictability that links the network connectivity of a firm to its expected excess returns. This prediction is testable in the data.[5]

Although the network structure is rather general, it is possible to obtain closed-form solutions for security prices. For simplicity, let us assume that all endowments D_t^i share the same diffusive component Y_t in (2). In the empirical section we let Y_t to be again stock-specific.

A Security prices

The equilibrium price-dividend (P/D) ratio of a security depends on two components: (*a*) the vector $\mathbf{H}$, capturing the global states of distress or non distress of all firms/trees in the network, and (*b*) the posterior probability of being in a boom state, p_t^h. Dividend jumps cause trees to enter a persistent distress or normalcy state, changing the state vector $\mathbf{H}_t$,[6] which in turn affects expected future dividend growth. The next Proposition describes P/D-ratios in terms of the P/D-ratios obtained in a complete information economy, in which the business cycle S is observable.

Proposition 1. *Let $P^i(\mathbf{H}_t)$ denote the price of the claim to the $i-th$ endowment, D_t^i. Let also $P_0^i(\mathbf{H}_t)$ and $P_1^i(\mathbf{H}_t)$ denote the full-information prices conditional on a boom or a recession, respectively. We have:*

$$\frac{P^i(\mathbf{H}_t)}{D_t^i} = p_t^h \frac{P_0^i(\mathbf{H}_t)}{D_t^i} + (1 - p_t^h)\frac{P_1^i(\mathbf{H}_t)}{D_t^i}, \tag{9}$$

Full-information prices $P_u^i(\mathbf{H}_t)$, $u = 0, 1$ are reported in (43) of Appendix A.

Intuitively, the agent estimates the P/D-ratios contingent on each business cycle state (i.e. for $S_t = 0$ or 1). These depend on the global state of the whole network. The incomplete information P/D ratio is a weighted average of the complete-information ratios, with weights given by the conditional beliefs of being in the corresponding state. To analyze the marginal contribution of the information channel and of the network structure, we remove both the characteristics and then reintroduce them sequentially.

1) Suppose agents observe S and there is no connectivity, so that jump intensities do not depend on $\mathbf{H}$. Any distress event leads to an expected increase in dividend growth for the affected asset, hence consumption growth in equilibrium, because a recovery is eventually foreseen. In this case, low future state prices (marginal utility) imply a *reduced desire to invest* in any risky assets in order to substitute consumption intertemporally, so that all P/D-ratios

[5] Additional implications can be derived for the behavior of aggregate consumption, interest rates and of the market prices of risk. A separate Appendix illustrates these properties, which are not the focus of this paper.

[6] $\mathbf{H}_t$ takes 2^N possible values, ranging from a combination where no tree/firm is in distress, $H_t^i = 0$, to one where all are in distress, $H_t^i = 1$, $i = 1, 2, \ldots, n + 1$.

drop. The higher the distress intensity λ and dividend share of the distressed tree, the more pronounced the negative spill-over effect, because the increment of expected consumption growth is maximal.

2) Adding network structure back, the trees which are connected to the distressed one will experience an increase of their distress jump intensity. Fears of distress contagion jeopardize consumption recovery, hence the agent may want to invest in securities that can hedge the lower perceived consumption growth. Procyclical assets are bad in this role, thus their demand will drop. These are firms which are able to spread their own distress risk, but relatively immune from distress contagion: the 'exogenously connected' firms. The intuition is simple. An exogenous firm lays in distress when aggregate consumption is systematically low, because of its ability to cause generalized distress. Its dividends are highly correlated with aggregate consumption. The situation is different for firms that are more endogenous to the shock and have modest or negative correlation with future aggregate consumption: their demand will rise, or it will diminish less, and so their P/D-ratios.

3) With incomplete information, each distress induces agents to update to a higher value their beliefs of a current recession. In recession all trees have higher distress risk and they are more interconnected. While the network-related contagion acts through the chain of connections, the perceived contagion acts systematically. The demand for securities flows towards less procyclical assets, those whose jump intensities are relatively insensitive to the business cycle: $\lambda(1, \mathbf{H}) \approx \lambda(0, \mathbf{H})$.

It is intuitive that the degree of exogeneity (i.e. whether a firm is $n = 1$ or $n = 4$ in Figure 2a) of an asset determines its location in the cross-section of P/D ratios. One may think that these characteristics are structural and related to either technological or contractual features that regulate the input/output links of the firm to others in the network. At the same time, they depend on the exposure of firms to a latent business cycle factor, that determines the extent to which inferring business cycle conditions from the cross-section of fundamentals affects their demand.

We will later suggest a global measure of dynamic centrality that accounts for these features.

B Risk premia

The equilibrium risk premium of the $i-$th security can be decomposed into a premium for diffusive risk, a premium for distress risk (μ_λ^i) and a premium for recovery risk (μ_η^i). The latter two involve a learning premium. These components are summarized in the following Proposition.

Proposition 2. *Let μ_t^i denote the equilibrium risk premium of the $i-th$ security. We have:*

$$\mu_t^i = \gamma\sigma_Y^2 + \mu_\lambda^i + \mu_\eta^i \tag{10}$$

$$\mu_\lambda^i = \sum_{j=1}^{n+1}(1-H_t^j)\left[\widehat{\lambda}^j(\mathbf{H}_t) - \widehat{R}^i(\mathbf{H}_t^{-j})\widehat{\lambda}_{rn}^j(\mathbf{H}_t)\right] \tag{11}$$

$$\mu_\eta^i = \sum_{j=1}^{n+1} H_t^j\left[\widehat{\eta}^j(\mathbf{H}_t) - \widehat{R}^i(\mathbf{H}_t^{+j})\widehat{\eta}_{rn}^j(\mathbf{H}_t)\right] \tag{12}$$

where $\mathbf{H}_t^{-j}$ ($\mathbf{H}_t^{+j}$) coincides with $\mathbf{H}_t$, except for firm j (not) in distress. The parameter $\widehat{\lambda}_{rn}^j$ ($\widehat{\eta}_{rn}^j$) is the risk-neutral intensity of distress (recovery) of firm j, reported in (47) of Appendix A.

$$\widehat{R}^i(\mathbf{H}_t^{-j}) = \frac{P^i(\mathbf{H}_t^{-j})}{P^i(\mathbf{H}_t)}, \quad \widehat{R}^i(\mathbf{H}_t^{+j}) = \frac{P^i(\mathbf{H}_t^{+j})}{P^i(\mathbf{H}_t)} \tag{13}$$

are the gross returns on security i when tree j has a distress or a recovery event, respectively. They are reported in expression (53) of Appendix A.

The first term, $\gamma\sigma_Y^2$, is the usual compensation for the diffusive risk of the common dividend component Y, independent of network connectivity.[7] Assume there is no connectivity and no incomplete information. The distress risk premium (10) becomes:

$$\mu_\lambda^i = \sum_{j=1}^{n+1}(1-H_t^j)\left[1-\theta_t^j R^i(S,\mathbf{H}_t^{-j})\right]\lambda^j(S,\mathbf{H}_t) \quad S=0,1. \tag{14}$$

The (full-information) risk-adjusted intensity of distress of firm j, $\lambda_{rn}^j(\,\cdot\,) = \theta_t^j\lambda^j(\,\cdot\,)$, is greater than the objective intensity λ^j: the parameter $\theta_t^j > 1$ can be interpreted as the risk adjustment per unit of (instantaneous) probability that the agent requires as compensation for the risk that the distress happens.[8] If the event materializes, security i responds with a gross returns R^i.[9] Thus the distress risk premium (14) is a weighted average of the risk adjusted returns on security i that would emerge had any tree a distress, with the likelihoods of distress as weights. As discussed in Section III.B, with no connectivity and full-information the distress jump of any tree j gives rise to lower P/D ratios for any security i, hence gross returns smaller than one. The equity premium (14) is at its highest.

When we allow for network connectivity the expression is still (14), but post-distress returns take into account the centrality of the asset in the network. In particular, if firm i_1 is less sensitive than firm i_2 to external distress, and more able to transfer its own distress to other firms in the network, then $R^{i_1}(S,\mathbf{H}_t^{-j}) \le R^{i_2}(S,\mathbf{H}_t^{-j})$, which implies that firm i_1 has a higher risk premium.

[7]In the following discussion we consider distress risk. The intuition for recovery risk is similar.
[8]It is smaller than one in case of recovery event. See Appendix A for details.
[9]The full-information counterpart of (13) are fully described in Appendix A, see equations (53) and (54).

Examples. Compare the disconnected network of Figure 1a to the 'Star' network of Figure 2a. To focus on network structure, assume that dividend payments are homogeneous across firms. In Figure 1a, trees' distress jumps are idiosyncratic, because their intensities are constant: $\lambda^j(u, \mathbf{H}_t) = \lambda^j$. In this framework the cross-section of equity premia is determined solely by the relative magnitude of distress intensities. For finite n, the lower the distress probability, the lower the risk premium. The reason is that in this case the firm pays dividends in low aggregate consumption states. Moreover, this risk is diversifiable as the number of firms, n, becomes arbitrarily large.

In a 'Star' network structure, as in Figure 2a, the situation changes. Firm 1 is central, because its distress jump increases all other distress intensities, but the converse is not true. Moreover, all other firms are disconnected among each other. Let us also assume that distress and recovery intensities coincide across noncentral firms, so that absent connectivity there is no cross-sectional variation of risk premia. In this context, firm 1 can be thought as a source of systemic risk in the network: according to the next Proposition, it has the highest risk premium as $n \to \infty$ and firms are not in distress, hence risk premia are directly comparable.

Proposition 3. *Consider a 'Star' network economy where a distress of Firm 1, the central firm, increases the other distress intensities by a factor k. Assume that Assumption 1 and Assumption 2 in Appendix A are satisfied. There exists a k^*, dependent on firm characteristics, such that as n gets arbitrarily large and $k > k^*$, Firm 1 has a higher risk premium than any noncentral Firm N, conditional on any present state $\mathbf{H}_t$ where both firms 1 and N are not in distress.*

The intuition is that Firm 1's distress translates into a systematic risk factor, by increasing the firms' chances of distress, and leading the economy towards a low aggregate consumption state. Since firms accrue to distress when Firm 1 lays in it, the latter displays highly cyclical pay-outs. The only chance for the central firm of being less or equally exposed to the trough it creates, is (a) to have superior recovery ability and pay-off normal dividends while most are still trapped in distress and the discount factor (marginal utility) is large, or (b) to cause immediate distress propagation, in which case all firms have the same loading on this factor. Concerning the last observation, one of the key assumptions of Proposition 3 is:[10]

$$k\lambda\left(1 - nk\lambda\Delta - n\eta\Delta\right) > \left(1 - n\lambda\Delta - n\eta\Delta\right)\lambda \tag{15}$$

for small Δ. The left-hand-side of (15) denotes the approximate probability of distress and permanence in this state over the next small time interval, for a noncentral firm, when Firm 1 is in distress, and the economy size is large.[11] This has to be larger than its counterpart when Firm 1 is not in distress, for the latter to be riskier. Intuitively, an excessive strength of distress propagation – i.e. a too large k – could not allow to distinguish significantly between central and

[10] See Assumption 2 in Appendix A. (15) reduces to expression (63).

[11] In which case the number of firms in distress or not is proportional to n.

noncentral firms, with regard to the correlation between their distress state and economic fundamentals (aggregate consumption, in our model). Indeed, with immediate propagation (infinite k) there cannot exist a state where the central firm is in distress and some other firm is not. On the other hand, insufficient propagation – i.e. $k < k^*$ – could also imply, for the opposite reason, economic fundamentals that are not significantly worse during Firm 1's distress compared to others'.

If we also restore incomplete information, we have to take into account how the assets dividend growth and its network connectivity behave relative to the business cycle. As noted previously, the less an asset is exposed – namely $\lambda(1, \mathbf{H}) \approx \lambda(0, \mathbf{H})$ in all states of $\mathbf{H}$ – the more it is valuable in a recession. With incomplete information its post-distress gross return – which amounts to the expression in brackets in (10) – is larger, and its risk premium is smaller.

IV The Failure of the Two-Fund Separation Property

The cross-sectional heterogeneity in the connectivity of different trees has immediate implications for the two-fund separation property of asset prices.

Proposition 4. *Consider a sequence of economies indexed by the total number of firms, n, where firms' characteristics satisfy Assumption 1 and Assumption 4 in Appendix A.*

If the network is symmetric, as in Figure 1, two fund-separation holds as $n \to \infty$: assets' risk-premia have an exact one-factor representation. If the network is asymmetrically connected, two fund separation does not hold. In the 'Star' form of Figure 2a, three fund-separation holds as $n \to \infty$: firms' risk-premia are linear combinations of the central firm's (firm 1) risk premium and of an additional risk factor.

In a completely homogeneous economy, where also dividend jumps are the same across trees in the limit of $n \to \infty$, it is well known that expected returns are equal to the sum of two components: (a) the risk free rate and (b) the marginal contribution of the asset to the variance of the market portfolio. Idiosyncratic risk is not priced. The main reason is that for $n \to \infty$ the market portfolio can diversify away firm-specific shocks, so that these will not bear any risk premium. Indeed this is the case for the disconnected network structure described in Figure 1a. In Figure 1b instead, where network connections are identical, shocks are only systematic, in that they have perfect correlation with shocks to the market portfolio when n grows arbitrarily large. While two-fund separation holds in these cases, it does not hold for the 'Star' network in Figure 2a. The intuition is simple: since firm 1 is dominant in the network, even for $n \to \infty$ the market portfolio is not able to diversify away its firm-specific risk. This result holds more generally: networks with a large cross-sectional dispersion in centrality do not satisfy the two fund separation property and firm-specific risk matters in equilibrium asset prices. Figure 2b reports a typical clustered economy. There are $\overline{n}$ connected central firms, each with its own 'Star' subnetwork. Noncentral firms are disconnected among them, but they relate heterogeneously to their 'Star'. The next Corollary generalizes Proposition 4 to this situation.

Corollary 1. *If the same assumptions of Proposition 4 hold, and the network is of the clustered 'Star' form of Figure 2b, with $\overline{n}$ 'Star' firms, $2\overline{n}+1-$fund separation holds as the number of noncentral firms in each subnetwork grows arbitrarily large and $\overline{n}$ remains finite.*

This result states that every central firm is a source of priced risk both because of its own idiosyncratic distress risk, and because of the idiosyncratic risk that is complementary to it. Noncentral firms are affected in distinct forms by their 'Star', even when 'Stars' are symmetrically connected, which creates independent forms of complementarity. Indeed, with additional assumptions – such as (15) in the one-star case – we could conjecture that the firm with the tightest links to its network – the most locally exogenous – has the larger expected return among the central firms, thus of all the economy, in light of Proposition 3. The intuition is that this firm suffers smaller consumption growth during its distress state, as the latter is more systematic for the economy. Similarly, it is reasonable to expect that the firm most affected by its 'Star' has larger risk premium among noncentral firms, as its non idiosyncratic ('Star'-related) distress is shared by more subnet peers on average, which makes it more correlated with consumption risk.

V An indicator of firm centrality

In network theory several parameters of connectivity have been proposed to describe the relation of nodes in a graph. Examples include the "Bonacich centrality" vector and the "Influence" vector. These parameters play an important role in static networks. In dynamic networks such as ours, we introduce a dynamic concept of centrality that describe the cash flow features that have direct asset pricing implications in our model. This measure is global in the sense that it uses information on the connectivity in the whole network, not just on the firms which are directly connected (neighboring firms). We define 'dynamic centrality' (or $\mathcal{DC}_{ij}^{\tau}$) of asset i relative to j over the horizon τ,the τ-deferred cross-correlation of distress of i and j:

$$\mathcal{DC}_{ij}^{\tau} = \frac{P[H_{t+\tau}^{j}=1, H_t^i=1] - P[H_{t+\tau}^{j}=1]P[H_t^i=1]}{\sqrt{P[H_{t+\tau}^{j}=1]P[H_t^i=1](1-P[H_{t+\tau}^{j}=1])(1-P[H_t^i=1])}} \tag{16}$$

$\mathcal{DC}_{ij}^{\tau}$ is the unconditional correlation between the events that i is in distress at time t and that j is in distress τ periods afterwards.[12] The joint probability at the numerator can also be written as

$$P[H_{t+\tau}^{j}=1, H_t^i=1] = P[H_{t+\tau}^{j}=1|H_t^i=1]P[H_t^i=1] \tag{17}$$

The above measure captures the distress causality between assets. In this sense, it is naturally related to the statistical concept of exogeneity. In a two-firm economy, if firm i has low $\mathcal{DC}_{ij}^{\tau}$ but

[12] Expressions for the probabilities are in Appendix A.

high $\mathcal{DC}^{\tau}_{ji}$ demands a lower expected return, because it is prone to be in distress when firm j has been in distress, but not the opposite. Firm i is 'actively' connected to the rest of the economy, and because of this property its distress status is strongly cyclical and influenced by the common factor. It should be noticed that since shocks in our economy propagate dynamically, depending on the extent of amplification or absorption due to the network structure, the previous measure of centrality has a time dimension and it should be thought as a term structure characteristics.

The measure in (16) is obtained using the transition probability of the global vector $\mathbf{H}$, to exploit information about all possible contemporaneous states of distress (or recovery). Since this is a multi-dimensional measure, one can obtain a synthetic metrics of centrality by calculating the (fundamental) size-weighted average of $\mathcal{DC}^{\tau}_{ij}$, $\forall j \neq i$. We define each asset's 'dynamic centrality' as

$$\overline{\mathcal{DC}}^{\tau}_{i} = \sum_{j=1, j\neq i}^{n} \left(\mathcal{DC}^{\tau}_{ij}\, w^{ij} - \mathcal{DC}^{\tau}_{ji}\, \frac{1}{w^{ij}} \right) \quad w^{ij} = \frac{\overline{D}^{i}(\exp(-J^{i}) - 1)}{\overline{D}^{j}(\exp(-J^{j}) - 1)} \tag{18}$$

Outgoing connections are scaled by a weight w^{ij}, equal to the average dividend loss of asset i upon distress as a fraction of the average dividend loss of asset j. Incoming connections are treated symmetrically. If dividends are an equal fraction of the aggregate endowment, no heterogeneous weighting is necessary. This measure of centrality implies that a high $\overline{\mathcal{DC}}^{\tau}_{i}$ indicates a firm that is central in the network: its shocks are mainly transfered to other firms but it is less exposed to other firms' shocks. It also implies a relatively large expected dividend loss upon distress relative to the others. A low $\overline{\mathcal{DC}}^{\tau}_{i}$ indicates the opposite case of a firm that is passively exposed to other firms' shocks. $\overline{\mathcal{DC}}^{\tau}$ is a global measure since it depends on the global characteristics of the vector $\mathbf{H}$ and not just on the connectivity between firm i and its neighboring firms.

Examples. Consider Figure 1a and Figure 1b. In the first case, $\overline{\mathcal{DC}}^{\tau}_{i} = 0\ \forall i$ since firms are disconnected. In the second case, the network is symmetric. If the symmetry is exact (i.e. firms also have the same size, transition intensities and jump size J^{i}), then also in this case $\overline{\mathcal{DC}}^{\tau}_{i} = 0$ for $\forall i$.

The case of Figure 2a is different. Firm 1 has the highest value of $\overline{\mathcal{DC}}^{\tau}$; all other firms for $j = 2, ..N$ have a value of $\overline{\mathcal{DC}}^{\tau}_{j}$ which depend on the extent to which a small shock to firm 1 implies a large loss of firm j's dividend.

A Implications of a Calibrated Economy

Based on previous intuition, the measure $\mathcal{DC}^{\tau}_{i}$ is positively related to expected returns. To help organize our empirical tests on live data, we calibrate and simulate the model to investigate this link.

We consider a network of $n = 10$ trees, aimed at capturing a set of stylized features of ten U.S. Industrial sectors. The calibration procedure is detailed in Appendix B. Risky assets are considered claims to sectors' output. We use this fictitious economy to confirm the main implication of the model, that we will later test in the empirical sections: larger dynamic centrality – $\mathcal{DC}^{\tau}_{i}$, with $\tau = 1$

month – should be associated to larger expected returns on average. Using closed-form expressions for security prices (9) and belief dynamics (6), we simulate 4500 years of monthly returns under the information set of the agent. We then find the sectors' dynamic centralities, $\mathcal{DC}_i^\tau$, $i = 1, \ldots, 10$. Panel 1 of Table IX relates the dynamic centrality measures to the cross-section of unconditional expected returns.

Insert Table IX

We find that a portfolio long the quartile of most exogenous assets, and short the least exogenous gains an average annualized return of 8.6 %, with a t-statistics of 14. While the relation is not linear, the results suggest that it is rational in a network to find a link between cash-flow connectivity and expected returns. As reported in Panel 2, the cross-sectional ranking of expected excess returns is increasing on average in the dynamic centrality measure. Indeed, the interpretation is of a risk premium.

The next Sections are devoted to testing this and other predictions on the data.

VI Empirical Analysis

A Data and Portfolios Formation

To study the potential role of structural network connectivity on expected returns, we merge two main datasets. The first updates the Fama-French (1992) sample of portfolio returns double sorted according to usual characteristics up to December 2007. We use these portfolios to populate the nodes in the network. The second dataset collects total cash-flow distribution for each node (portfolio) in the network which we use to estimate firm connectivity. These two components are jointly used to run extended Fama-MacBeth regressions controlling for dynamic centrality.

Portfolio Characteristics and Returns. We follow Fama and French (1992) and populate a double sorted set of 10×10 portfolios according to market beta and size (the "β-size" portfolios thereafter). We update portfolios in June of each year, and for each we compute value-weighted returns from July to June of next year. Our data consists of monthly stock returns on all firms listed on NYSE, AMEX and NASDAQ, with accounting data reported in the COMPUSTAT database from 1963 to 2007. Portfolios are formed in June of each year (year t). Betas of individual stocks are computed from a time series regression of excess returns on the market excess return, for 24 to 60 of the months preceding June of year t (included). The market equity value of individual stocks used in the size-sorting is recorded in June of year t. The monthly market returns and risk-free rate are from K. French.[13] Portfolios comprise a minimum of 333 and a maximum of 4659 firms. As in Fama and French (1992), we use the time series of returns of a given portfolio type to compute its beta, as the sum of the slopes in a time-series regression of excess returns on contemporaneous and 1-month lagged excess market returns. In the Fama-McBeth regressions to

[13] 'F-F_Benchmark_Factors_Monthly' of K. French's website.

follow, the portfolio beta is assigned to each individual stock belonging to the portfolio in a given year, and each stock is assigned its book and market equity values available in December of year $t-1$. Thus portfolio construction and asset pricing tests are both designed to avoid any use of contemporaneous or look-ahead bias.

Dividends and Cash-Flow Distribution. Once stocks are assigned to portfolios, we collect quarterly data on their cash-flow distributions, namely total dividends (Compustat DVT), repurchases of common and preferred stock (Compustat PRSTKC) and the redemption value of the preferred stock (Compustat PSTKRV), as well as the number of shares outstanding. As for beta, size, and book-to-market, we avoid including forward looking information by using only previous fiscal year cash-flow distributions. We follow the procedure outlined in Menzly, Santos, and Veronesi (2004), Hansen, Heaton, and Li (2002) and Bansal, Dittmar, and Lundblad (2002), to build the assets cash-flow series that take into account shares repurchases and redemptions, and that are consistent with value-weighted holdings of stocks in portfolios. Let $j = 1, \ldots, 100$ denote a given portfolio, Ω_t^j the collection of stocks in that portfolio at month t, and V_t^j its market value. Let t coincide with a portfolio updating date (July of each year).

- At time t, for each stock $i \in \Omega_t^j$ we find the number of shares θ_t^i that satisfies the value-weighting condition.

- During the quarter running from t to $t+1$, total cash-flows accruing to portfolio j are:

$$D_{t,t+1}^j = \sum_{i \in \Omega_t^j} \theta_t^i \frac{DVT_{t+1}^i + PRSTKC_{t+1}^i - (PSTKRV_{t+1}^i - PSTKRV_t^i)}{N_t^i},$$

- If repurchases or redemptions occur for stock i during the quarter, the number of shares held is updated in percentage of the total repurchase/redemption, excluding potential new issues:

$$\theta_{t+1}^i = \theta_t^i \frac{N_t^i - [PRSTKC_{t+1}^i - (PSTKRV_{t+1}^i - PSTKRV_t^i)]/P_{t+1}^i}{N_t^i}.$$

  The numerator is the total number of shares outstanding at the beginning of next quarter before new issues.

- At $t+1$, the ex-dividend market value of portfolio j is $V_{t+1}^j = \sum_{i \in \Omega_{t+1}^j} \theta_{t+1}^i P_{t+1}^i$. The quarterly total return on the portfolio is $R_{t+1}^j = (V_{t+1}^j + D_{t,t+1}^j - V_t^j)/V_t^j$. Ω_{t+1}^j coincides with Ω_t^j, until date $t+4$, when the portfolio composition is updated and the procedure repeated.

- As in Menzly, Santos, and Veronesi (2004): we assume an initial investment in portfolio j, V_0^j, corresponding to the market capitalization of the portfolio per US capita: $V_0 = \sum_{i \in \Omega_0^j} N_0^i P_0^i / pop(0)$.[14] $pop(0)$ is the US population at time 0, June 1953; we assume that

[14] Without loss of generality we have multiplied this figure by 100.

the consumption flow C_t is the per-capita US total consumption expenditure of non-durables plus services, as reported, already deflated and deseasoned, by National Income and Product Accounts (NIPA).

We deseason cash-flow series using a four quarter trailing moving average. The model is estimated on the cash-flows of 16 beta-size sorted portfolios.

The value weighted returns on the portfolios include cash-flow distributions in proportion the holdings in each stock. Individual stocks returns also include cash-flow distributions.

B Model Estimation

We estimate the structural parameters of the model to test the role of the network structure on asset prices. We relax two assumptions which were introduced for theoretical tractability. First, we allow the diffusive dividend growth component Y to be stock specific, with drift and diffusion coefficients μ^i and σ^i, $i = 1, \ldots, n+1$. Second, the jump component of logarithmic dividend growth, J^i can assume different values for distress and recovery jumps, with J_1^i and J_2^i respectively. We calibrate the transition intensities of the common factor to the transition characteristics of NBER recession cycles. Then, we use dividend data to estimate the parameters of the dividend process and compute the structural measure of dynamic centrality proposed in the tehoretical section that matches the connectivity found in the data. Each sorted portfolio is assigned to a node in the network and provides an aggregation mechanism to reduce measurement error and create the dispersion in expected excess returns that we want to study. Finally, we conduct Fama-French (1992) style regressions and study the extent to which dynamic centrality is a helpful characteristic to understand the cross-section of expected excess returns.

Step 1. (Transition Intensities). We set the transition intensities of the common factor S, k_h and k_l to values consistent with the characteristics of NBER recession cycles. We use the following system of exactly identified unconditional moment conditions:

$$\begin{aligned} \pi(S=0) &= 0.6893 \\ \pi(S=0)\frac{1}{k_h^2} + \pi(S=1)\frac{1}{k_l^2} &= \frac{56.1818}{12} \end{aligned}$$

where $\pi(S=0) = k_l/(k_h + k_l)$ is the stationary probability of a non recession and 0.6893 is the fraction of time spent outside recession in the last 156 years. The left hand side of the second expression is the average unconditional duration of a cycle, either recession or not. Notice that $\int_0^\infty \exp(-k_l s) s \, ds = 1/k_l^2$ is the expected duration of a non recession (simarly for a recession). We obtain $k_l = 1.2616$ and $k_h = 0.3911$.

Step 2. (Dividend Process). To estimate the connectivity in the network and compute a measure of dynamic centrality for each firm, we use firm-level dividend data aggregated at the portfolio level. To address the dual goal of reducing estimation error and parameter propagation, we populate each node of the network with firms obtained from the double sorts on market beta and size. We estimate the process on portfolio pairs, using a methodology similar to the Simulated Maximum Likelihood method of Brandt and Santa-Clara (2002), as used also in David and Veronesi (2012). Then, we aggregate to obtain the average level of dynamic centrality for each portfolio. We estimate the dividend flow in the network based on observations of portfolio dividend pairs $\{\hat{D}^i_t, \hat{D}^j_t\}$, for all distinct portfolios i and j. The simulation step is needed because we don't observe either the distress and the events, or the posterior beliefs that determine their intensities in the agent's perception. We obtain parameters' standard errors using the asymptotic distribution of the estimates. Details of the procedure are in Appendix B.

Step 3. (Dynamic Centrality). Using statistically significant parameters, we compute the dynamic centrality measure (18) for each portfolio. Table XI reports a graphical representation of the network structure for the 4×4 beta-size sorted portfolios. The length of each circle is associated to the magnitude of portfolio's dynamic centrality, which aggregates the information about pairwise network links: the deferred distress correlations in (16). Links departing from portfolio i are the $\mathcal{DC}_{ij}$s, distress correlations obtained preconditioning on i's past distress, where j is some other portfolio. Links arriving to portfolio i are the $\mathcal{DC}_{ji}$s, that precondition on j's past distress. Stronger causations correspond to solid lines.

Insert Table XI

C Empirical Results

C.1 Characteristics of Sorted Portfolios

We analyze the characteristics of the 100 market beta-size sorted portfolios, obtained with the approach of Fama and French (1992). Panel 1 of Table I reports portfolio betas obtained from time series regressions on the whole sample (the "post ranking betas"). Betas range from 0.63 to 2.03, hence display significant heterogeneity. The variation across the beta dimension shows that ex-post portfolio betas are consistent with the ex-ante betas of the ranked stocks. Variation across the size dimension confirms the well known fact that beta is inversely correlated with size. We have also explored the book-to-market equity (BE/ME) dimension, by stratifying the same portfolios into BE/ME deciles. According to Panel 2 of Table I, post ranking betas are less heterogeneous across the BE/ME dimension, with both value and growth stocks characterized by higher betas. The non-monotonicity of betas across BE/ME summarizes the well know inconsistency of the CAPM with the value premium.

Insert Table I

Table II reports portfolios average monthly returns. Average returns display a strong decreasing pattern along the size dimension, with figures decreasing monotonically from 3.3% to 0.1% per month in Panel 1 as size increases from the first to the 10th decile. The pattern along the beta dimension is much weaker. Not only the range is smaller (0.9%-1.8%) but a consistently increasing pattern of returns in betas is present only from the 5-th beta decile onwards. While book-to-market sorting produces a weaker heterogeneity in average portfolio returns, it gives rise to an increasing returns pattern in BE/ME, as expected (Panel 2).

Insert Table II

Table III reports in Panel 1 the conditional ($\overline{\mathcal{DC}}_i^{\tau}$, $\tau = 1m$) dynamic centrality measure of the portfolios. Dynamic centrality is roughly decreasing in size, although the correlation is not perfect, as the pattern is not always monotonic even considering this relatively coarse stratification. No clear pattern emerges with respect to market beta. Panel 2 reports the average value-weighted returns of 5 dynamic centrality-sorted portfolios, corresponding to the quintiles of their distribution with respect to the measure. Average returns of dynamic centrality sorted portfolios are increasing in the dynamic centrality. We form a portfolio that is long the fifth dynamic centrality quintile and short the first; then we regress its returns on the contemporaneous Fama-French factors: market excess return, book-to-market (hml) and size (smb) factors. As shown in Panel 2b, the positive return expected from buying centrality are not explained by the Fama-French factors: for a portfolio average return equal to 1.7% monthly we obtain a statistically significant alpha of 1.26% monthly. This suggests that network structure is indeed relevant for asset prices.

Insert Table III

C.2 Fama-MacBeth Regression

We now turn to estimate the price of risk associated to the centrality factor. We follow closely the Fama and French (1992) approach of using the whole cross section of individual stock returns available at a given month. For monthly observations comprised between July of year t and June of year $t+1$, we assign to a given stock its size (ME) and book-to-market as reported in December of year $t-1$ and the post ranking beta of the portfolio to which it belongs. The centrality matching procedure mimics the beta matching procedure, so that between July and June of a given year a stock is assigned the centrality of its portfolio. At each month, we run a cross-sectional regression of excess returns on beta, size, book-to-market and centrality. We report time-series averages of coefficients and the corresponding t-statistics, computed using time-series standard deviations of coefficients. Table IV reports results for various combinations of the explanatory factors.

Insert Table IV

The centrality price of risk is positive in all specifications, meaning that more exogenous stocks gain higher expected returns. Dynamic centrality is significant at standard confidence levels, although its negative correlation with book-to-market and positive correlation with size affect its t-statistics when we run a joint regression with all factors, leaving it still within the significance level. Controlling for all factors, an increase of one standard deviation in the centrality of a stock implies a 0.18% gain in monthly expected return.[15]

Centrality measures for beta-size sorted portfolios (Table III) imply centrality premium components which range from 0 to approximately 0.7% monthly. Concerning beta, size and book-to-market the findings on our sample broadly reproduce those of Fama-French (1992). The slopes with respect to BE/ME and size are, respectively, positive and negative and both strongly significant across all specifications. The evidence of a positive market beta disappears after one controls for size.

C.3 EIV and Shanken's Correction

Both the CAPM betas and the centrality measures employed in cross-sectional regressions are estimates, rather than realizations of the true factors, hence prone to estimation error. As noted in Shanken (1996a,b), while asymptotic consistency of the slope coefficients is guaranteed by the consistency of the estimators, Fama-MacBeth standard errors underestimate the true slopes' standard errors. This issue is partially addressed by assigning post-ranking portfolios betas and centrality to individual stocks. Shanken (1996) proposes an asymptotic correction for the slopes' t-statistics of CAPM betas to account for the errors-in-variables problem. Since this correction is not readily adaptable to the centrality measures, we rely on Monte-Carlo simulations. Table V reports summary statistics of simulated Fama-MacBeth slopes and t-statistics, obtained by repeatedly sampling from the distribution of the measurement error and running the regressions on each sample. Details of the procedure are in Appendix B.

The positive relation between expected returns and centrality is relatively robust to measurement error. The median and mode slope coefficients are larger than the empirical value reported in Table IV. The positive fifth percentile of the slope distribution shows that, after correcting for errors in variables, a negative value is very unlikely. The Fama-MacBeth t-statistics has a median across repetitions of 1.98, and a mode of 2.07.[16] The positive beta-expected returns relation, on the other hand, does not appear significant regardless of measurement error: the median t-statistics is 1.24, the mode is 1.40, and its empirical probability of being smaller than 1.96 is 93%.

Insert Table V

D Network structure and cross-sectional momentum

The model can be used to help understanding a second important effect in asset pricing: cross-sectional momentum. In the previous Sections we have tested whether an *aggregate* measure of

[15] The figure refers to the sample standard deviation of centrality across the beta-size sorted portfolios.

[16] Note that this experiment does not provide the small sample distribution of the slopes and t-statistics, since we are only sampling parameters from the asymptotic distribution of first stage regressors, hence accounting only for measurement error in these variables.

stock 'centrality' is a priced factor in the cross section of expected returns. $\mathcal{DC}_i^\tau$ is an aggregate indicator because it summarizes information on the distress connectivity of stock i with each element of the cross-section. In doing so, it takes into account the strength of the connectivity, in terms of average cash-flow distress of which i is responsible relative to the whole. In this Section we dissect $\mathcal{DC}_i^\tau$ and exploit the information of the network structure at a more localized level. Our empirical strategy is aimed at exploring whether our network structure can help explain the results in Menzly and Ozbas (2011), who find that returns of an industry are positively related to past returns of industries connected by a supplier or customer relationship. They use data from input-output BEA tables and argue that predictability emerges for these firms when analyst coverage is scarce and past connected returns help resolve uncertainty. We use our model-implied notion of dynamic cash-flow centrality to distinguish the role played by different stocks. We use the previously estimated network and, without changing any of its characteristics, we document the extent to which it is consistent with their results.

D.1 Long-short portfolio procedure

Cross-sectional momentum cannot be generated simply by the existence of a network structure linking different firms. Without informational frictions, any shock would have instantaneous implications on the cross-section of expected returns. However, if there are informational frictions (such as limited and segmented analyst coverage) then dividend shocks to a firm may be transfered to prices of other connected firms with a lag, but in ways that is consistent with a network structure. This may generate a phenomenon that has been described in the literature as cross-sectional momentum. In what follows, we will presume the existence of such informational frictions.

Consider a firm i with low centrality $\overline{\mathcal{DC}}_i$. Dividend shocks to firm i will tend to lag those of firms that are upstream to i. Unless agents have perfect information, it is possible that stock i's expected returns are correlated with past returns of upstream stocks. When we test for this prediction we account for two features: i) past returns of a connected tree are relevant in proportion to the 'amount of shock' that the tree transfers: how bad has the distress been or how good the recovery. This channel affects discount factors through aggregate consumption, hence it affects the expected returns of both trees involved. ii) Past returns of connected stocks may not matter if the strength of even the closest connection is weak, that is, incoming correlation $\mathcal{DC}_{ji}^{1m\cdot}$ is small.[17]

We work with the network estimated on beta-size sorted portfolios, focusing on the set Ω_i of the four upstream portfolios that are most connected to portfolio i, namely the portfolios j with the highest 'transferring' correlation $\mathcal{DC}_{ji}^{1m}$. The average 'transferring' correlation among these four then measures how strongly i's fundamentals are determined by its peers.[18] We sort portfolios into 4 bins of average 'transferring' correlation, to address point ii) above: once we condition on similar strength of connectivity, we can disentangle the predicting effect of the type of shock being propagated. Θ_j denotes the $j-$th connection strength bin, $j = 1, \ldots, 4$. For each beta-size sorted

[17] $\mathcal{DC}_{ji}^{1m\cdot}$ is an incoming correlation from the point of view firm i.

[18] Formally, the highest average 'transferring' correlation for i is $\frac{1}{4}\sum_{j\in\Omega_i}\mathcal{D}_{ji}^{1m\cdot}$

portfolio i, we construct a signal, $\mathcal{S}^i_{t-1}$, which depends positively on the past performance of the most connected stocks, and takes into account the relative amplitude of transfered shocks.[19] After assigning the signal $\mathcal{S}^i_{t-1}$ to each stock in portfolio i, we sort stocks in the same bin θ_j according to their signal, and form a value-weighted portfolio long the last quintile and short the first of the distribution.

Controlling for the strength of the connectivity, stocks whose mostly connected peers did worse last month should gain lower present returns. Table VI reports summary statistics of the returns on the long-short portfolios and results obtained when we regress them in time on contemporaneous Fama-French factors.

Insert Table VI

Reading the Table row-wise we condition on connectivity strength. Average returns of long-short portfolios are positive in all bins, strongly significant in the first and the last, and weakly in the third and fourth. A positive and significant α emerges from the time-series regressions, meaning that the portfolio performance cannot be fully attributed to exposition to Fama-French factors. This test supports the presence of cross-sectional momentum originating from stocks that most determine other stocks' fundamentals in the network structure. Reading the table from the top to the bottom, we compare portfolios in increasing order of connectivity strength. Past returns of connected stocks should matter more at the bottom. The intuition is confirmed from the second bin onwards, where average portfolio returns are increasing. The first bin is an exception, which in our opinion is due to the substantial presence of stocks with small market capitalization: the time-series regression confirms that this portfolio is highly exposed to size.

D.2 Fama-MacBeth Regression

To test for cross-momentum between connected stocks controlling for informational frictions, we also apply the Fama-MacBeth procedure. We test whether the factor $\mathcal{S}^i_{t-1}$ in (19) is priced in the cross-section of expected returns, and the sign of its price of risk. As highlighted earlier, Menzly and Ozbas (2011) find evidence of cross-momentum among stocks with modest analyst coverage, for which technological connections help resolve the uncertainty due to incomplete diffusion of information. This is in principle consistent with our model, as we have seen in Section II that the cross-sectional learning, induced by shocks to trees, has a wider impact in situations of high

[19] Formally, $\mathcal{S}^i_{t-1}$ is defined as:

$$\mathcal{S}^i_{t-1} = \sum_{u\in\Omega_i} \omega_u \left(\sum_{z\in u} \omega^u_z r^z_{t-1} \right) \quad \omega_u = \frac{\overline{D}^u(\exp(-J^u)-1)}{\sum_{v\in\Omega_i} \overline{D}^v(\exp(-J^v)-1)} \tag{19}$$

Index z denotes the individual stocks in portfolio u, which is one of the four beta-size sorted portfolio most connected to i. ω^u_z is the value weight of stock z in portfolio u, and r^z_{t-1} its one-month past return. ω_u is the average dividend loss upon distress of portfolio u in percentage of the overall loss of portfolios most connected to i. While returns on individual stocks are value-weighted inside their portfolio, returns of portfolios most connected to i are weighted in proportion to the relative amplitude of the fundamental shock that the tree portfolio can transfer.

uncertainty about the economic state ($p_t^h \approx 0.5$). Stocks that are highly actively connected (exogenous) react with a shock of the same sign, while endogenous stocks under react or react contrarily.

We work with the network estimated on beta-size sorted portfolios. We merge this sample with the I/B/E/S dataset, from which we retrieve the number of monthly forecasts about stocks' earnings-per-share, from June 1983 to July 2008. For each stock i we build the variable I_t^i, which takes value h if in month $t-1$ the stock has 3 or more analyst covering the firm, and l otherwise. We then interact the signal $\mathcal{S}_{t-1}^i$ with an analyst coverage dummy and with a size dummy, as informational frictions likely depend on firm size. In other words, we construct the eight factors:

$$x_t^{i,cove,size} = \mathcal{S}_{t-1}^i \mathbf{1}(I_t^i = cove)\mathbf{1}(s_t^i = size); \quad cove = h, l \quad size = s, m_1, m_2, b \tag{20}$$

where $\mathbf{1}(X_t = x)$ denotes a dummy variable which takes value 1 if $X_t = x$ and 0 otherwise. s_t^i is stock i's quartile at time t within the market equity distribution. We then run the cross-sectional regressions:

$$r_t^i = a_t + \gamma_t' x_t^{i,size,cove} + \beta_t' C_t + \epsilon_t. \tag{21}$$

C_t are the traditional risk factors including beta, book-to-market; we also include short-term reversal (REV) and medium term continuation (MTCONT), as constructed in Menzly and Ozbas (2011). Table VII reports time series average and standard errors of regression coefficients for various specifications.

Insert Table VII

Evidence of cross-momentum is confined to stocks in the first size-quartile with low analyst coverage, for which the price of risk is positive and statistically significant. This is consistent with an information-based motivation of cross-sectional momentum. We also find some weaker evidence of cross-sectional reversal among bigger firms with larger analyst coverage, which, contrarily to cross-momentum's, is not fully robust to the inclusion of control factors.

VII Conclusions

We study a dynamic economy where network links among firms' cash-flows generate cross-sectional predictability of returns. Our framework builds on two main features: (a) network connectivity, which is introduced by making dividend jump intensities dependent on states of other trees and on a latent common factor; (b) incomplete information about the systematic or idiosyncratic nature of a shock. We explore the link between firms' degree of centrality in the network and the cross-sectional dispersion of expected returns. Highly 'exogenous' firms, which actively determine the propagation of fundamental shocks in the economy, are pronouncedly procyclical and should gain higher expected returns. We introduce an empirical measure of dynamic network centrality, and we use data on dividend distributions at the portfolio level to test these predictions. Consistent with the prediction, we find evidence of a positive price of risk for the centrality factor. We also employ the cash-flow connectivity structure to investigate cross-sectional momentum, that is, to which extent past returns of connected firms can predict present returns. Consistent with the model's predictions, we find evidence of cross-momentum for firms with a small centrality and narrow analyst coverage.

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Appendix A

Proof of Lemma 1. We use the notation $\mathcal{F}_t$ to denote the full information set, i.e. the filtration generated by the Brownian motion Z_t, the distress (or not) status vector $\mathbf{H}_t$, and the business cycle state S_t. $\mathcal{F}_t^{x,Y}$ denotes the restricted information set of the agent, or observation filtration, generated only by $\mathbf{H}_t$ and Z_t. Informative signals about S_t consists of negative ($dH_t^i = H_{t-}^i - H_t^i = 1$) or positive ($dH_t^i = -1$) dividend growth jumps of tree i, $i = 1, \ldots, n+1$, because jump intensities depend on S_t. We have also referred to negative (positive) jumps as transitions to (out of) distress. The continuous dividend component Y_t conveys no information about the latent business cycle. With respect to $\mathcal{F}_t$, the intensity of the compound Poisson process H_t^i is

$$\lambda_t^{H^i} = -H_t^i \eta^i(S_t, \mathbf{H}_t) + (1 - H_t^i)\lambda^i(S_t, \mathbf{H}_t^i) \tag{22}$$

We need the following Lemma, whose proof is an adaptation of Theorem 19.1 in Lipster and Shyriaev (2001).

Lemma 2. *Any $\mathcal{F}_t^{x,Y}-$ martingale X_t admits the representation:*

$$X_t = X_0 + \int_0^t \sum_{i=1}^{n+1} f_s^{H^i}(dH_s^i - \widehat{\lambda}_s^{H^i} ds)$$

where adapted processes $f_t^{H^i}$ satisfies the integrability conditions in Theorem 19.1 of Lipster and Shyriaev (2001).

It follows from Theorem 18.3 in Lipster and Shyriaev (2001) that, with respect to $\mathcal{F}_t^{x,Y}$ (incomplete information), expression (22) becomes

$$\widehat{\lambda}_t^{H^i} = -H_t^i[p_t^h \eta^i(0, \mathbf{H}_t^i) + \eta^i(1, \mathbf{H}_t^i)(1 - p_t^h)] + (1 - H_t^i)[\lambda^i(0, \mathbf{H}_t^i)p_t^h + \lambda^i(1, \mathbf{H}_t^i)(1 - p_t^h)] \tag{23}$$

By Lemma 9.2 in Lipster and Shyriaev(2001) we have that, for $j = 0, 1$, the random process

$$y_t^j = \mathbf{1}(\lambda^i(S, \mathbf{H}_t) = \lambda^i(j, \mathbf{H}_t)) - \mathbf{1}(\lambda^i(S, \mathbf{H}_0) = \lambda^i(j, \mathbf{H}_0)) - \int_0^t [-\mathbf{1}(\lambda^i(S, \mathbf{H}_s) = \lambda^i(j, \mathbf{H}_s))k_j + \mathbf{1}(\lambda^i(S, \mathbf{H}_s) = \lambda^i(j^c, \mathbf{H}_s))k_{j^c}]ds \tag{24}$$

is an $\mathcal{F}_t-$martingale, where j^c denotes the complement of j. Taking conditional expectations with respect to $\mathcal{F}_t^{x,Y}$ in the definition of y_t^j, we obtain:

$$p_t^h = p_0^h + \int_0^t [-p_s^h k_j + p_s^h k_{j^c}]ds + \mathbb{E}[y_t^j | \mathcal{F}_t^{x,Y}] \tag{25}$$

We can now apply the martingale representation theorem in Lemma 2 above to the martingale $\mathbb{E}[y_t^j|\mathcal{F}_t^{x,Y}]$ and identify stochastic integrands as in Lipster and Shyriaev (2001), Theorem 19.5. We end up with the representation given in the Lemma. This ends the proof.

Proof of Proposition 1. Given initial conditions $\mathbf{H}_0$ and x_0^i, $i = 1, \ldots, n+1$, since the negative (positive) jump intensity is zero if a tree is already in distress (in non distress), $H_t^i = 1$ ($H_t^i = 0$), we can think of the persistent dividend component x in (2) as a two-state Markov chain, with states $\overline{x}^i$ and $\underline{x}^i$, where $\overline{x}^i > \underline{x}^i$. This chain is in $\overline{x}^i$ if $H_t^i = 0$ and in $\underline{x}^i$ otherwise. The relation between persistent dividend states and log dividend growth jump size J^i is:

$$J^i = \log \frac{\overline{x}^i}{\underline{x}^i} \tag{26}$$

In what follows we drop functional arguments for the intensities λ and η when no confusion may arise.

According to the optimality conditions for the representative agent, the equilibrium state price density, ξ_t, is:

$$\xi_t = e^{-\delta t} Y_t^{-\gamma} \left(\sum_{i=1}^{n+1} x_t^i \right)^{-\gamma} \tag{27}$$

On the other hand, for any security price P_t^i adapted to $\mathcal{F}_t^{x,Y}$, including the risk-less bond, the discounted price process $(P_t^i \xi_t + \int_0^t \xi_s D_s^i ds)$ is a martingale. Ito's lemma then implies that ξ_t must also obey:

$$\xi_t = \exp\left(-\int_0^t (r_s + \frac{\kappa_s^2}{2}) ds - \int_0^t \kappa_s dZ_s + \int_0^t \sum_{i=1}^{n+1} \widehat{\lambda}_s^{H^i} (1-\theta_s^i) ds + \int_0^t \sum_{i=1}^{n+1} -\log(\theta_s^i) \mathrm{sgn}(H_t^i) dH_s^i \right) \tag{28}$$

where $\mathrm{sgn}(H_t^i) = -1$ if $H_t^i \le 0$ and $\mathrm{sgn}(H_t^i) = 1$ if $H_t^i > 0$. Furthermore, θ_t^i is the market price of event risk for tree i - distress risk, if tree i is in not in distress, i.e. $H_t^i = 0$, recovery risk if tree i is in distress, i.e. $H_t^i = 1$ - κ_t is the market price of diffusive risk, and

$$\widehat{\lambda}_t^{H^i} = H_t^i \widehat{\eta}^i + (1 - H_t^i) \widehat{\lambda}_t^i.$$

By applying Ito's lemma for jump-diffusion processes to (28), we obtain:

$$d\xi_t = -\xi_t r_t dt - \xi_t \kappa dZ_t + \xi_t \left[\sum_{i=1}^{n+1} (\theta_s^i - 1)(-\mathrm{sgn}(H_t^i) dH_t^i - \widehat{\lambda}_t^{H^i}) \right] \tag{29}$$

By Ito's lemma for jump-diffusion processes applied instead to (27), we obtain the alternative representation:

$$\begin{aligned} d\xi_t = -\delta\xi_t - \gamma\mu\xi_t dt + \frac{1}{2}\gamma(\gamma+1)\sigma^2\xi_t dt + \xi_t \sum_{i=1}^{n+1} \Bigg[(1-H_t^i) \frac{\left[(\underline{x}^i + \sum x_{t-})^{-\gamma} - (\overline{x}^i + \sum x_{t-})^{-\gamma}\right]}{(\overline{x}^i + \sum x_{t-})^{-\gamma}} \widehat{\lambda}_t^i \\ + H_t^i \frac{\left[(\overline{x}^i + \sum x_{t-})^{-\gamma} - (\underline{x}^i + \sum x_{t-})^{-\gamma}\right]}{(\underline{x}^i + \sum x_{t-})^{-\gamma}} \widehat{\eta}_t^i \Bigg] - \gamma\xi_t\sigma dZ_t + \\ \xi_t \sum_{i=1}^{N} \Bigg[(1-H_t) \frac{\left[(\underline{x}^i + \sum x_{t-})^{-\gamma} - (\overline{x}^i + \sum x_{t-})^{-\gamma}\right]}{(\overline{x}^i + \sum x_{t-})^{-\gamma}} (dH_t^i - \widehat{\lambda}_t^i) + H_t^i \frac{\left[(\overline{x}^i + \sum x_{t-})^{-\gamma} - (\underline{x}^i + \sum x_{t-})^{-\gamma}\right]}{(\underline{x}^i + \sum x_{t-})^{-\gamma}} (-dH_t^i - \widehat{\eta}_t^i) \Bigg] \end{aligned} \tag{30}$$

$\sum x_{t-}$ denotes the sum of persistent dividend components across trees, excluding i, an instant before the jump of i takes place. Matching the coefficients of expression to (30) to those of expression (29), we obtain the equilibrium interest rate and market prices of risk:

$$r_t = \delta + \gamma\mu_Y - \frac{1}{2}\gamma(\gamma+1)\sigma_Y^2 + \sum_{i=1}^{N} \left\{ H_t^i \left[1 - \left(\frac{\overline{x}^i + \sum x_{t-}}{\underline{x}^i + \sum x_{t-}} \right)^{-\gamma} \right] \widehat{\eta}_t^i + \right. \tag{31}$$

$$\left. (1 - H_t^i) \left[1 - \left(\frac{\underline{x}^i + \sum x_{t-}}{\overline{x}^i + \sum x_{t-}} \right)^{-\gamma} \right] \widehat{\lambda}_t^i \right\} \tag{32}$$

$$\kappa_t = \gamma\sigma_Y \tag{33}$$

$$\theta_t^i = H_t^i \left(\frac{\overline{x}^i + \sum x_{t-}}{\underline{x}^i + \sum x_{t-}} \right)^{-\gamma} + (1 - H_t^i) \left(\frac{\underline{x}^i + \sum x_{t-}}{\overline{x}^i + \sum x_{t-}} \right)^{-\gamma} \quad i = 1, 2, \ldots n+1 \tag{34}$$

Let $\mathbf{H}_t$ denote the current vector of distress (or not) state for the trees, Y_t the current continuous dividend component, and p_t^h the current posterior probability of a boom. The price of the claim to the $i-$th endowment processes, $P^i(\mathbf{H}_t)$, is a function of these variables. We denote by $P_1^i(\mathbf{H}_t)$ and $P_0^i(\mathbf{H}_t)$ the full-information prices conditional on a recession or a boom, respectively. Given the equilibrium state-price density ξ_t as in (27), the absence of arbitrage opportunities implies that $P^i(\mathbf{H}_t)$ reads:

$$\begin{aligned} P^i(\mathbf{H}_t) &= \frac{1}{\xi_t}\mathbb{E}\left[\int_t^\infty \xi_s Y_s x_s^i ds \middle| \mathcal{F}_t^{x,Y}\right] \\ &= \frac{Y_t}{(\sum_{i=1}^{n+1} x_t^i)^{-\gamma}}\mathbb{E}\left[\mathbb{E}\left[\int_t^\infty e^{-a(s-t)} x_s^i \left(\sum_{j=1}^{n+1} x_s^i\right)^{-\gamma} ds \middle| \mathcal{F}_t\right] \middle| \mathcal{F}_t^{x,Y}\right] \qquad (35) \\ &= \frac{Y_t}{(\sum_{i=1}^{n+1} x_t^i)^{-\gamma}} \left[p_t^h V(0,\mathbf{H}_t) + (1-p_t^h)V(1,\mathbf{H}_t)\right] \qquad (36) \end{aligned}$$

where

$$a = \delta - \mu_Y(1-\gamma) + \frac{\sigma_Y^2}{2}(1-\gamma)\gamma.$$

and

$$V^i(u,\mathbf{H}_t) = \mathbb{E}\left[\int_t^\infty e^{-a(s-t)} x_s^i \left(\sum_{j=1}^{n+1} x_s^i\right)^{-\gamma} ds \middle| \mathcal{F}_t, S_t = u\right] \quad u = 0,1. \qquad (37)$$

Equation (35) follows from the independence of Y_t and x_t and from the law of iterated expectations. We need to compute $V(u,\mathbf{H}_t)$. The process

$$\int_0^t e^{-as} x_s^i \left(\sum_{j=1}^{N} x_s^i\right)^{-\gamma} ds + e^{-at}V^i(u,H_t) \qquad (38)$$

is an $\mathcal{F}_t-$martingale, therefore its 'drift' component must vanish. Remember that $\lambda^{H^i}(u,\mathbf{H}_t)$, $u=0,1$ denoted the full-information intensity of the distress state for tree i: its intensity of recovery if the tree is in distress, its intensity of distress otherwise:

$$\lambda^{H^i}(u,\mathbf{H}_t) = (1-H_t^i)\lambda^i(u,\mathbf{H}_t) + H_t^i\eta^i(u,\mathbf{H}_t) \quad u = 0,1$$

We assume that $a > 0$. to guarantee finite asset prices. Applying Ito's lemma to (38), taking conditional expectations, and imposing the martingale property, we end up with the following system of equations:

$$\begin{bmatrix}0\\0\end{bmatrix} = \left(\begin{bmatrix} -a-\sum_{j=1}^{n+1}\lambda^{H^j}(0,\mathbf{H}_t) & 0 \\ 0 & -a-\sum_{j=1}^{n+1}\lambda^{H^j}(1,\mathbf{H}_t)\end{bmatrix} + \begin{bmatrix}-k_h & k_h \\ k_l & -k_l\end{bmatrix}\right)\begin{bmatrix}V^i(0,\mathbf{H}_t)\\ V^i(1,\mathbf{H}_t)\end{bmatrix} + \\ \begin{bmatrix}\sum_{j=1}^{n+1}\lambda^{H^j}(0,\mathbf{H}_t)V^i(0,\mathbf{H}_t \pm j) \\ \sum_{j=1}^{n+1}\lambda^{H^j}(1,\mathbf{H}_t)V^i(1,\mathbf{H}_t \pm j)\end{bmatrix} + \begin{bmatrix} x_t^i\left(\sum_{j=1}^N x_t^j\right)^{-\gamma} \\ x_t^i\left(\sum_{j=1}^N x_t^j\right)^{-\gamma}\end{bmatrix} \qquad (39)$$

The current distress (or not) vector state, $\mathbf{H}_t$, moves to the combination $\mathbf{H}_t + j$ if tree j recovers from a distress, or to $\mathbf{H}_t - j$ if it experiences a distress. $V^i(u,\mathbf{H}_t \pm j)$, $u = 0,1$, denotes the function (37) conditional to one of these two states. Since there are 2^{n+1} possible states for $\mathbf{H}_t$,[20] and the function $V^i(u,\mathbf{H}_t)$ must be solved jointly for all states, it is clear that (39) is a system of $2 \times 2^{n+1}$ linear equations. Eventually we can stack functions $V^i(u,H)$, for all distress (or not) combinations H, into a $2 \times 2^{n+1}$ vector $\mathbf{V}^i$, where $V^i(0,H)$ and $V^i(1,H)$ are contiguous.

[20]Of course not all of them are mutually reachable, because at most one of the trees can fall in distress or recover at some time instant.

The vector $\mathbf{V}^i$ that solves the system (39) is

$$\mathbf{V}^i = (\mathbf{a} + \mathbf{A}^{\mathbf{H}})^{-1}\mathbf{C}^i \tag{40}$$

where

$$\mathbf{a} = \mathrm{diag}_{2\times 2^{n+1}}(a) \tag{41}$$

$$\mathbf{A}^{\mathbf{H}} = \mathrm{diag}_{2^{n+1}}\left(\begin{bmatrix} \sum_{j=1}^{n+1} \lambda^{H^j}(0,H) & 0 \\ 0 & \sum_{j=1}^{n+1} \lambda^{H^j}(1,H) \end{bmatrix} + \begin{bmatrix} -k_h & k_h \\ k_l & -k_l \end{bmatrix}\right) - \mathbf{L}^{\mathbf{H}} \tag{42}$$

$\mathbf{L}^{\mathbf{H}}$ is a $(2\times 2^{n+1})\times 2\times 2^{n+1}$ matrix whose entry $(2h-1, 2u-1)$ is the intensity, in the boom state, of the tree that needs to distress or recover to reach the $u-$th state of $\mathbf{H}_t$ from the $h-$th. The entry $(2h, 2u)$ is the same intensity contingent to the recession state. Therefore $\mathbf{A}^{\mathbf{H}}$ is the joint Markov transition matrix of the state variables $(\mathbf{H}, S)$. $\mathbf{C}^i$ is the $2\times 2^{n+1}$ vector whose entry $2h-1$ is $x_t^i\left(\sum_{j=1}^N x_t^j\right)^{-\gamma}$ in the $h-$th state of $\mathbf{H}_t$: it is the persistent dividend paid in that state, discounted by the marginal rate of intertemporal substitution. $V^i(0,\mathbf{H}_t)$ and $V^i(1,\mathbf{H}_t)$ are the appropriate entries of $\mathbf{V}^i$, corresponding to the current state $\mathbf{H}_t$. Finally

$$P_1^i(\mathbf{H}_t) = \frac{Y_t}{(\sum_{i=1}^{n+1} x_t^i)^{-\gamma}} V^i(1,\mathbf{H}_t) \tag{43}$$

$$P_0^i(\mathbf{H}_t) = \frac{Y_t}{(\sum_{i=1}^{n+1} x_t^i)^{-\gamma}} V^i(0,\mathbf{H}_t) \tag{44}$$

Proof of Proposition 2. We report expressions for $\lambda_{rn}^i(S,\mathbf{H}_t)$, $\widehat{\lambda}_{rn}^i(\mathbf{H}_t)$, $\eta_{rn}^i(S,\mathbf{H}_t)$ and $\widehat{\eta}_{rn}^i(\mathbf{H}_t)$:

$$\lambda_{rn}^i(S,\mathbf{H}_t) = \lambda^i(S,\mathbf{H}_t)\theta_t^i \tag{45}$$

$$\eta_{rn}^i(S,\mathbf{H}_t) = \eta^i(S,\mathbf{H}_t)\theta_t^i \tag{46}$$

$$\widehat{\lambda}_{rn}^j(\mathbf{H}_t) = p_t^h\lambda_{rn}^i(0,\mathbf{H}_t) + (1-p_t^h)\lambda_{rn}^i(1,\mathbf{H}_t) \tag{47}$$

$$\widehat{\eta}_{rn}^j(\mathbf{H}_t) = p_t^h\eta_{rn}^i(0,\mathbf{H}_t) + (1-p_t^h)\eta_{rn}^i(1,\mathbf{H}_t) \tag{48}$$

$$\theta_t^i = H_t^i\left(\frac{\overline{x}^i + \sum x_{t-}}{\underline{x}^i + \sum x_{t-}}\right)^{-\gamma} + (1-H_t^i)\left(\frac{\underline{x}^i + \sum x_{t-}}{\overline{x}^i + \sum x_{t-}}\right)^{-\gamma} \qquad i = 1,2,\dots n+1 \tag{49}$$

Recall from (29) that $\theta_t^i - 1$ is the market price of tree i's risk of dividend growth jumps, either distress or recoveries, depending on i's present state. $\lambda^i\theta_t^i$ and $\eta^i\theta_t^i$ are the risk-adjusted intensities of distress and recoveries. The agent behaves risk-neutrally after this adjustment. $\widehat{\lambda}_{rn}^j(\mathbf{H}_t)$ and $\widehat{\eta}_{rn}^j(\mathbf{H}_t)$ are their partial information counterparts.

The risk premium of the security reads:

$$\mu_t^i = \mathbb{E}\left[\frac{dP^i(\mathbf{H}_t)}{P^i(\mathbf{H}_t)}\middle|\mathcal{F}_t^{x,Y}\right] + \frac{D_t^i}{P^i(\mathbf{H}_t)} - r_t \tag{50}$$

In order to find its expression, we apply Ito's lemma to the martingale $M_t^i = \xi_t P^i(\mathbf{H}_t) + \int_0^t \xi_s D_s^i ds$, taking into account expression (29) for the state-price density. We obtain

$$dM_t^i = \xi_t D_t^i dt + \xi_t P^i(\mathbf{H}_t) m_t^i dt - \xi_t P^i(\mathbf{H}_t) r_t dt - \xi_t P^i(\mathbf{H}_t)\kappa\sigma_Y dt - \xi_t P^i(\mathbf{H}_t)(\kappa - \sigma_Y) dZ_t +$$
$$\sum_{j=1}^{n+1} H_t^j\left[\theta_t^j\xi_t P^i(\mathbf{H}_t^{+j}) - \xi_t P^i(\mathbf{H}_t)\right](-dH_t^j) + \sum_{j=1}^{n+1}(1-H_t^j)\left[\theta_t^j\xi_t P^i(\mathbf{H}_t^{-j}) - \xi_t P^i(\mathbf{H}_t)\right]dH_t^j \tag{51}$$

m_t^i denotes security i's instantaneous expected return $\mathbf{E}[dP^i/P^i|\mathcal{F}_t^{x,Y}]$. $\mathbf{H}^{-j}$ ($\mathbf{H}^{+j}$) is the distress (or not) vector to which the present state $\mathbf{H}_t$ jumps in case tree j had a distress (recovery). Dividing both sides by $\xi_t P^i(\mathbf{H}_t)$, taking expectations and recalling that they vanish, we obtain:

$$\mu_t^i = m_t^i + \frac{D_t^i}{P^i(\mathbf{H}_t)} - r_t = \kappa\sigma_Y - \sum_{j=1}^{n+1} H_t^j \left[\theta_t^j \frac{P^i(\mathbf{H}_t^{+j})}{P^i(\mathbf{H}_t)} - 1\right]\widehat{\eta}_t^j - \sum_{j=1}^{n+1}(1-H_t^j)\left[\theta_t^j \frac{P^i(\mathbf{H}_t^{-j})}{P^i(\mathbf{H}_t)} - 1\right]\widehat{\lambda}_t^j \tag{52}$$

Since $\kappa = \gamma\sigma_Y$, the RHS of (52) coincides with the expression reported in the Proposition. Taking into account expression (36) for security prices and the dynamics of the posterior belief in (6), the gross return on security i, triggered by a distress or recovery of tree j, reads explicitly:

$$\frac{P^i(\mathbf{H}_t^{-j})}{P^i(\mathbf{H}_t)} = \left(\frac{\underline{x}^j + \sum x_{t-}}{\overline{x}^j + \sum x_{t-}}\right)^{\gamma} \left(\frac{\left(p_t^h \frac{\lambda^j(0,\mathbf{H}_t)}{\widehat{\lambda}^j(\mathbf{H}_t)}, (1-p_t^h)\frac{\lambda^j(1,\mathbf{H}_t)}{\widehat{\lambda}^j(\mathbf{H}_t)}\right)\cdot(V^i(0,\mathbf{H}_t - j), V^i(1,\mathbf{H}_t - j))'}{(p_t^h, 1-p_t^h)\cdot(V^i(0,\mathbf{H}_t), V^i(1,\mathbf{H}_t))'}\right) \tag{53}$$

$$\frac{P^i(\mathbf{H}_t^{+j})}{P^i(\mathbf{H}_t)} = \left(\frac{\overline{x}^j + \sum x_{t-}}{\underline{x}^j + \sum x_{t-}}\right)^{\gamma} \left(\frac{\left(p_t^h \frac{\eta^j(0,\mathbf{H}_t)}{\widehat{\eta}^j(\mathbf{H}_t)}, (1-p_t^h)\frac{\eta^j(1,\mathbf{H}_t)}{\widehat{\lambda}^j(\mathbf{H}_t)}\right)\cdot(V^i(0,\mathbf{H}_t + j), V^i(1,\mathbf{H}_t + j))'}{(p_t^h, 1-p_t^h)\cdot(V^i(0,\mathbf{H}_t), V^i(1,\mathbf{H}_t))'}\right) \tag{54}$$

Functions V^i are those of of expression (37). The full-information premium (14) follows after setting $p_t^h = 0$ (conditional to a current recession) or $p_t^h = 1$ (conditional to a current boom) in (52).

Expressions of probabilities in distress correlations (16). Unconditional marginal probabilities are obtained from the steady-state distribution of the joint markov chain that governs $(\mathbf{H}, S)$. The transition matrix of this process is the matrix $\mathbf{A}^{\mathbf{H}}$ in (41). Therefore the $2^{n+1}\times 2$ vector of steady state probabilities, for each possible combination of $\mathbf{H}$ and state of the business cycle S, solves:

$$\pi' = \pi' \exp\left(-\mathbf{A}^{\mathbf{H}}\right) \tag{55}$$

We obtain π numerically by iterating equation (55) until convergence is reached. To obtain $P[H_{t+\tau}^j = 1]$ ($= P[H_t^j = 1]$) and $P[H_t^i = 1]$ we sum the entries of π over all the states of $\mathbf{H}$ where j is in distress. The conditional probability $P[H_{t+\tau}^j = 1|H_t^i = 0]$ is given by the standard solution of the Chapman-Kolmogorov equations for the process $(\mathbf{H}, S)$:

$$P[H_{t+\tau}^j = 1|H_t^i = 0] = \mathbf{I}_i' \exp\left(-\mathbf{A}^{\mathbf{H}}\tau\right)\mathbf{I}_j$$

$\mathbf{I}_j$ is a $2^{n+1}\times 2$ vector with ones for the combinations of $\mathbf{H}$ where tree j is in distress and zero otherwise. $\mathbf{I}_i$ is similarly defined. When we work with transition matrices of pairs of trees, as in the empirical application, the expressions are the same, after selecting the appropriate transition matrix $\mathbf{A}^{\mathbf{H}}$.

Proof of Proposition 3. Let $\mathbf{H}_t$ denote a distress (or not) state where firms 1 and N re not in distress ($H_t^1 = H_t^N = 0$). Let Δ be a small time interval. We denote by $\mathcal{P}_S^i(\Delta, \mathbf{H}_t)$ the full-information price in state $\mathbf{H}_t$ of the claim on dividends of firm i paid until time $t+\Delta$, evaluated at time t: the dividend strip that expires in $t+\Delta$.

With full-information, the price of the dividend strip can be found along the lines of the proof of Proposition 1. We have

$$\frac{\mathcal{P}_S^i(\Delta, \mathbf{H}_t)}{Y_t} = \frac{1}{\left(\sum_{j=1}^{n+1} x_t^j\right)^{-\gamma}} \mathbb{E}\left[\int_t^{t+\Delta} e^{-a(s-t)} x_s^i \left(\sum_{j=1}^{n+1} x_s^i\right)^{-\gamma} ds \middle| \mathcal{F}_t\right] \tag{56}$$

$$= \overline{\mathbf{1}}'(\mathbf{H}_t)\int_t^{t+\Delta} \exp(-(\mathbf{a}+\mathbf{A}^H)(s-t))ds\,\mathbf{C}^i \tag{57}$$

$\exp(\,\cdot\,)$ denotes the matrix exponential. $\overline{\mathbf{1}}(\mathbf{H}_t)$ is $2^{n+1}-$dimensional column vector with 1 in the entry corresponding to state $\mathbf{H}_t$ and zeros otherwise. Matrices $\mathbf{a}$, $\mathbf{A}^H$, and vector $\mathbf{C}^i$ are reported in (41) and thereafter. Since Δ is small, we can also write:

$$\begin{aligned}\frac{\mathcal{P}_S^i(\Delta,\mathbf{H}_t)}{Y_t} &\approx \overline{\mathbf{1}}'(\mathbf{H}_t)\exp(-(\mathbf{a}+\mathbf{A}^H)\Delta)\,\mathbf{C}^i\Delta \\ &\approx \overline{\mathbf{1}}'(\mathbf{H}_t)\mathbf{A}\,\mathbf{C}^i\Delta\end{aligned} \tag{58}$$

where $\mathbf{A} = [I - (\mathbf{a}+\mathbf{A}^H)\Delta)]$. We can think of the stock price (an infinite maturity dividend strip) as an infinite sum of prices of forward-start dividend strips:

$$\begin{aligned}\frac{P_S^i(\mathbf{H}_t)}{Y_t} &= \frac{1}{\left(\sum_{u=1}^{n+1}x_t^u\right)^{-\gamma}}\mathbb{E}\left[\sum_{j=0}^{\infty}e^{-a(t_j-t)}\left(\sum_{u=1}^{n+1}x_{t_j}^u\right)^{-\gamma}\frac{\mathcal{P}_S^i(\Delta,\mathbf{H}_{t_j})}{Y_{t_j}}\middle|\mathcal{F}_t\right] \\ &= \frac{1}{\left(\sum_{j=1}^{n+1}x_t^j\right)^{-\gamma}}\left[\sum_{j=0}^{\infty}\overline{\mathbf{1}}'(\mathbf{H}_t)\exp(-(\mathbf{a}+\mathbf{A}^H)(t_j-t))\int_{t_j}^{t_j+\Delta}\exp(-(\mathbf{a}+\mathbf{A}^H)(s-t_j))ds\,\mathbf{C}^i\right] \\ &\approx \frac{1}{\left(\sum_{j=1}^{n+1}x_t^j\right)^{-\gamma}}\left[\overline{\mathbf{1}}'(\mathbf{H}_t)\mathbf{A}\,\left(\mathbf{C}^i+\mathbf{A}\mathbf{C}^i+\mathbf{A}\mathbf{A}\mathbf{C}^i+\ldots\right)\Delta\right]\end{aligned}$$

where $t_0 = t$ and $t_j - t_{j-1} = \Delta$.

For simplicity we drop the explicit dependence on S from intensity parameters. We can think of S as an additional argument of the aggregate state of distress (or not) $\mathbf{H}$. We use the notation $x^i(\mathbf{H}_t)$ to denote the persistent dividend component paid by firm i in state $\mathbf{H}_t$.

As we are going to consider limiting behaviors as the number of firms n grows unboundedly, we impose the following assumptions.

Assumption 1. *Dividends are homogeneous across assets, and they are deterministic functions of the economy size n:*

$$x_t^i(H) = \begin{cases}\overline{f}(n) & \text{if } H_t^i = 0 \\ \underline{f}(n) & \text{if } H_t^i = 1\end{cases} \qquad i = 1,\ldots n \tag{59}$$

Moreover

$$\lim_{n\to\infty}\left(\frac{\sum_{j=1}^n x_t^j(H)}{\sum_{j=1}^n x_t^j(\mathbf{H}_t)}\right)^{-\gamma}\frac{x_t^i(H)}{x_t^i(\mathbf{H}_t)} = c(H,\mathbf{H}_t) \tag{60}$$

with $0 < c(H,\mathbf{H}_t) < \infty$, for all possible states H.

We have emphasized the dependence of the limits on the particular state of aggregate distress for the economy. For simplicity we drop the dependence on n from the $x_t^i(\,\cdot\,)$.

Assumption 2. *Let $\mathbf{H^1}$ denote the collection of states where firm 1 is in distress, and $\overline{\mathbf{H^1}}$ the states where it is not. Then:*

i) $\lambda^j(\mathbf{H}^1) = k\lambda$ *and* $\lambda^j(\overline{\mathbf{H}^1}) = \lambda$, $j = 2,3,\ldots,n$, *with* $k > 1$.

ii) $\eta^j(\overline{\mathbf{H}^1}) = \eta^j(\mathbf{H}^1) = \eta$.

Intensity parameters depend on economy size n, in such a way that total distress and recovery risk are bounded as $n\to\infty$:

$$\begin{aligned}\lim\nolimits_{n\to\infty} n\lambda &= K^\lambda < \infty \\ \lim\nolimits_{n\to\infty} n\eta &= K^\eta < \infty\end{aligned} \tag{61}$$

which implies

$$\lim_{n\to\infty} \lambda = \lim_{n\to\infty} \eta = 0 \tag{62}$$

The centrality parameter k satisfies the condition:

$$1 - K^{\lambda}(k+1)\Delta - K^{\eta}\Delta > 0 \tag{63}$$

for small Δ.
For simplicity we drop the dependence on n from λ and η.

Assumptions 1 simplifies the asymptotic behavior of dividend shares while insuring finite price-dividend ratios, thus allowing us to focus on distress connectivity, where the network structure relies. Assumption 2 serves two purposes: condition (61) guarantees finite asymptotic asset prices and risk premia, while condition (63) is a balance condition which, as discussed in the text, guarantees that firm 1 is more exposed to its distress risk, by limiting the extent of distress propagation. The dividend homogeneity assumption, which is formalized as

$$x^j(\mathbf{H}^1) = \overline{x} \qquad x^j(\overline{\mathbf{H}^1}) = \underline{x}, \quad j = 1, 2, 3, \ldots, n, \tag{64}$$

marginalizes the role of share size and let network connectivity drive the heterogeneity in risk premia.

The risk premium of the claim to the $i-$th firm is obtained from (14) of the text, after joining distress and recovery risk in a single expression:

$$\mu_t^i = \gamma\sigma^2 + \sum_{j=1}^{n} \tilde{\lambda}_n^j(\mathbf{H}_t)\left(1 - \theta_n^j(\mathbf{H}_t)\frac{P^i(\mathbf{H}_t^{\pm j})}{P^i(\mathbf{H}_t)}\right) \tag{65}$$

or

$$-\left[\mu_t^i - \gamma\sigma^2 - \sum_{j=1}^{n} \tilde{\lambda}^j\right] P^i(\mathbf{H}_t) = \sum_{j=1}^{n} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)P^i(\mathbf{H}_t^{\pm j}) \tag{66}$$

where $\tilde{\lambda}^i = H_t^i\eta^i + (1 - H_t^i)\lambda^i$. $\theta(\mathbf{H}_t) - 1$ is the market price for the distress or recovery risk of firm j reported in (49). $P^i(\mathbf{H}_t^{\pm j})$ is the price to which security i jumps immediately after the distress or recovery of the j-th tree. Using expression (59) to represent $P^i(\mathbf{H}_t^{\pm j})$, it is convenient to restate the RHS of (66) as:

$$\mathcal{R}^i = \sum_{j=1}^{n} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\left[\overline{\mathbf{1}}'(\mathbf{H}_t^{\pm j})\sum_{u=0}^{\infty}\exp(-(\mathbf{a} + \mathbf{A}^H)(t_u - t))\mathbf{A}\mathbf{C}^i\right] \tag{67}$$

Let $\mathcal{A}^i = \mathbf{A}\mathbf{C}^i$, with $\mathcal{A}^i(H)$ denoting the entries of $\mathcal{A}^i$ corresponding to state H. We also have

$$\overline{\mathbf{1}}'(\mathbf{H}_t^{\pm j})\sum_{u=0}^{\infty}\exp(-(\mathbf{a} + \mathbf{A}^H)(t_u - t)) = \sum_{u=0}^{\infty} e^{-a(t_u - t)} Prob\left(\mathbf{H}_{t_u} \middle| \mathbf{H}_t^{\pm j}\right) \tag{68}$$

where $Prob\,(\cdot)$ is the row vector of transition probabilities from time t to t_u conditional on state $\mathbf{H}_t^{\pm j}$ at time t.

We need the following two lemmas:

Lemma 3. *For any state H where firms 1 and N are both in distress or they are both not in distress, $\mathcal{A}^N(H) - \mathcal{A}^1(H) = 0$.*

Proof. When firms 1 and N are both in distress in state H we have:

$$\begin{aligned}\mathcal{A}^N(H)-\mathcal{A}^1(H)=&\sum_{j\in\mathcal{ND}(H)}k\lambda\Delta\left[C^N(H^{-j})-C^1(H^{-j})-\left(C^N(H)-C^1(H)\right)\right]\\&-\sum_{\substack{j\in\mathcal{D}(H)\\ j\neq 1,N}}\eta\Delta\left[C^N(H)-C^1(H)-\left(C^N(H^{+j})-C^1(H^{+j})\right)\right]+(1-a)(C^N(H)-C^1(H))\\-&\underbrace{\eta\Delta\left[C^N(H)-C^1(H)-\left(C^N(H^{+1})-C^1(H^{+1})\right)\right]}_{1}-\underbrace{\eta\Delta\left[C^N(H)-C^1(H)-\left(C^N(H^{+N})-C^1(H^{+N})\right)\right]}_{2}\end{aligned} \tag{69}$$

We have used the notation $C^i(H)$ to denote the entry of vector C^i that corresponds to state H. H^{+j} (H^{-j}) denotes the state reached from H when firm j recovers (has a distress). $\mathcal{D}(H)$ ($ND(H)$) denotes the collection of firm in (non) distress in state H^1. Using the homogeneous dividends assumption *iii*), we have $C^N(H)-C^1(H)=0$, $C^N(H^{-j})-C^1(H^{-j})=0$, $C^N(H^{+j})-C^1(H^{+j})=0$, $j\neq 1,N$, while terms 1 and 2 in (69) are opposite, so that $\mathcal{A}^N(H)-\mathcal{A}^1(H)=0$.

When firms 1 and N are both not in distress H we have:

$$\begin{aligned}\mathcal{A}^N(H)-\mathcal{A}^1(H)=&\sum_{\substack{j\in\mathcal{ND}(H)\\ j\neq 1,N}}\lambda\Delta\left[C^N(H^{-j})-C^1(H^{-j})-\left(C^N(H)-C^1(H)\right)\right]\\&-\sum_{j\in\mathcal{D}(H)}\eta\Delta\left[C^N(H)-C^1(H)-\left(C^N(H^{+j})-C^1(H^{+j})\right)\right]+(1-a)(C^N(H)-C^1(H))\\+&\underbrace{\lambda\Delta\left[C^N(H^{-1})-C^1(H^{-1})-\left(C^N(H)-C^1(H)\right)\right]}_{1}+\underbrace{\lambda\Delta\left[C^N(H^{-N})-C^1(H^{-N})-\left(C^N(H)-C^1(H)\right)\right]}_{2}\end{aligned} \tag{70}$$

Using the homogeneous dividends assumption *iii*), we have $C^N(H)-C^1(H)=0$, $C^N(H^{-j})-C^1(H^{-j})=0$, $C^N(H^{+j})-C^1(H^{+j})=0$, $j\neq 1,N$, while terms 1 and 2 in (70) are opposite, so that $\mathcal{A}^N(H)-\mathcal{A}^1(H)=0.\square$

Lemma 4. *Consider two states, $H^1\in\mathbf{H^1}$ and $\overline{H}^1\in\overline{\mathbf{H^1}}$, identical in all components except 1 and N: in H^1 firm N is not in distress, while in in $\overline{H}^1$ firm N is in distress. If in the initial state $\mathbf{H}_t$ both firms are not in distress, then:*

$$\lim_{n\to\infty}\left[kProb\left(\mathbf{H}_{t_u}=H^1\middle|\,\mathbf{H}_t\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H^1}\middle|\,\mathbf{H}_t\right)\right]\lambda\Delta\geq 0, \tag{71}$$

and there exists a $k^*(\lambda,\eta,k_h,k_l)$ such that, for $k>k^*$

$$\lim_{n\to\infty}\left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\,\mathbf{H}_t\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H^1}\middle|\,\mathbf{H}_t\right)\right]\lambda\Delta\leq 0 \tag{72}$$

for any H^1, with $t_u\geq t$.

Furthermore:

$$\begin{aligned}&Prob\left(\mathbf{H}_{t_u}=H^1\middle|\,\mathbf{H}_t\right)\left[\mathcal{A}^N(H^1)-\mathcal{A}^1(H^1)\right]-Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\,\mathbf{H}_t\right)\left[\mathcal{A}^1(\overline{H}^1)-\mathcal{A}^N(\overline{H}^1)\right]=\\&\left[kProb\left(\mathbf{H}_{t_u}=H^1\middle|\,\mathbf{H}_t\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\,\mathbf{H}_t\right)\right]\lambda\Delta\left[\sum_{\substack{j\in\mathcal{ND}(H^1)\\ j\neq N}}\left(C^N(H^{1-j})-C^1(H^{1-j})-\left(C^N(H^1)-C^1(H^1)\right)\right)\right.\end{aligned} \tag{73}$$

$$- \left(C^N(H^1) - C^1(H^1)\right)\Big] + \Big[Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t\right)\Big] \Big[(1-a)(C^N(H^1) - C^1(H^1)) +$$

$$\left. \sum_{\substack{j\in\mathcal{D}(H^1)\\ j\neq 1}} \eta\Delta\left(\left(C^N(H^{1+j}) - C^1(H^{1+j})\right) - \left(C^N(H^1) - C^1(H^1)\right)\right) - \eta\Delta\left(C^N(H^1) - C^1(H^1)\right)\right] \tag{74}$$

Proof. For simplicity we adopt the following notation, limited to this proof: $p_t^{t_u}(H) = Prob\left(\mathbf{H}_{t_u} = H \middle| \mathbf{H}_t\right)$. Similarly to the proof of Lemma 3, H^{+j} (H^{-j}) denotes the state reached from state H when firm j recovers (has a distress). In the same fashion, $H^{+j_1-j_2+j_3}$, for instance, denotes the state reached from state H after a recovery of firm j_1, then a distress of firm j_2, then a recovery of firm j_3. We decompose the time interval $t_u - t$ into n^Δ subintervals of arbitrarily small length Δ, with n^Δ an arbitrarily large integer such that $\Delta n^\Delta = t_u - t$. We are going to use repeatedly the following facts:
Properties:

1. For small Δ and $H \in \mathbf{H^1}$:

$$Prob\left(\mathbf{H}_{t_u} = H \middle| \mathbf{H}_{t_u-\Delta} = H\right) \approx e^{-[\text{num}(\mathcal{D}(H))\eta + \text{num}(\mathcal{ND}(H))k\lambda]\Delta} \tag{75}$$

$$Prob\left(\mathbf{H}_{t_u} = H^{-j} \middle| \mathbf{H}_{t_u-\Delta} = H\right) \approx 1 - e^{-k\lambda\Delta} \tag{76}$$

$$Prob\left(\mathbf{H}_{t_u} = H^{+j} \middle| \mathbf{H}_{t_u-\Delta} = H\right) \approx 1 - e^{-\eta\Delta} \tag{77}$$

$$\tag{78}$$

   for some firm j. $\text{num}(\mathcal{D}(H))$ ($\text{num}(\mathcal{ND}(H))$) is the number of firms that are (not) in distress in H. If $H \in \overline{\mathbf{H^1}}$ the expression $k\lambda$ is replaced by λ.

2 For any state H and $k > 1$:

$$\lim_{n\to\infty} k e^{-[\text{num}(\mathcal{D}(H))\eta + \text{num}(\mathcal{ND}(H))k\lambda]\Delta} \geq e^{-[\text{num}(\mathcal{D}(H))\eta + \text{num}(\mathcal{ND}(H))\lambda]\Delta} \tag{79}$$

   because

$$\lim_{n\to\infty} k e^{-[\text{num}(\mathcal{D}(H))\eta + \text{num}(\mathcal{ND}(H))k\lambda]\Delta} - e^{-[\text{num}(\mathcal{D}(H))\eta + \text{num}(\mathcal{ND}(H))\lambda]\Delta} \approx \lim_{n\to\infty} k\left[1 - (\text{num}(\mathcal{D}(H))\eta + \right.$$
$$\left.\text{num}(\mathcal{ND}(H))k\lambda)\,\Delta\right] - \left[1 - (\text{num}(\mathcal{D}(H))\eta + \text{num}(\mathcal{ND}(H))\lambda)\,\Delta\right] = \lim_{n\to\infty}(k-1) - (k-1)\text{num}(\mathcal{D}(H))\eta\Delta$$
$$- (k^2-1)\text{num}(\mathcal{ND}(H))\lambda\Delta \geq (k-1)\left[1 - K^\eta\Delta - (k+1)K^\lambda\Delta\right] > 0 \tag{80}$$

   for small Δ. The last equality in (80) follows from assumption (63).

3. $\text{num}(\mathcal{ND}(H^1)) = \text{num}(\mathcal{ND}(\overline{H}^1))$,[21] and similarly for the number of firms in distress, therefore we don't distinguish between these expressions.

4. Excluding firms 1 and N, the set of (non) distressed trees in H^1 and $\overline{H}^1$ coincide.

We apply the Chapman-Kolmogorov equations to express $p_t^{t_u}(H^1)$ and $p_t^{t_u}(\overline{H^1})$ in terms of one-step backward transition probabilities $p_t^{t_u-\Delta}(\,\cdot\,)$. Identifying all states from which H^1 and $\overline{H^1}$ can be reached, by the fact that in small time Δ at most one recovery or distress event can occur, we can write:

$$\left[k p_t^{t_u}(H^1) - p_t^{t_u}(\overline{H^1})\right]\lambda\Delta = \Bigg[\underbrace{p_t^{t_u-\Delta}(H^{1+1})k\left(1-e^{-\lambda\Delta}\right) - p_t^{t_u-\Delta}(H^{1+1})\left(1-e^{-\lambda\Delta}\right)}_{1}$$

[21]Similarly for any state reached after a sequence of common events, such as $H^{1+j_1-j_2+j_3}$ and $\overline{H}^{1+j_1-j_2+j_3}$, $j_1, j_2, j_3 \neq 1, N$.

$$+\underbrace{p_t^{t_u-\Delta}(H^{1-N})k\left(1-e^{-\eta\Delta}\right)-p_t^{t_u-\Delta}(H^{1-N})\left(1-e^{-\eta\Delta}\right)}_{2}$$

$$+p_t^{t_u-\Delta}(H^1)ke^{-\left[\text{num}(\mathcal{D}(H^1))\eta+\text{num}(\mathcal{ND}(H^1))k\lambda\right]\Delta}$$

$$-p_t^{t_u-\Delta}(\overline{H^1})e^{-\left[\text{num}(\mathcal{D}(H^1))\eta+\text{num}(\mathcal{ND}(H^1))\lambda\right]\Delta}$$

$$+\sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}} p_t^{t_u-\Delta}(H^{1+v})k\left(1-e^{-k\lambda\Delta}\right)-\sum_{\substack{v\in\mathcal{D}(\overline{H}^1)\\ v\neq N}} p_t^{t_u-\Delta}(\overline{H}^{1+v})\left(1-e^{-\lambda\Delta}\right)$$

$$\left.+\sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}} p_t^{t_u-\Delta}(H^{1-v})k\left(1-e^{-\eta\Delta}\right)-\sum_{\substack{v\in\mathcal{ND}(\overline{H}^1)\\ v\neq 1}} p_t^{t_u-\Delta}(\overline{H}^{1-v})\left(1-e^{-\eta\Delta}\right)\right]\lambda\Delta \tag{81}$$

Since terms 1 and 2 in (81) are nonnegative because $k>1$, we can write:

$$\left[kp_t^{t_u}(H^1)-p_t^{t_u}(\overline{H^1})\right]\lambda\Delta=\left[p_t^{t_u-\Delta}(H^1)ke^{-\left[\text{num}(\mathcal{D}(H^1))\eta+\text{num}(\mathcal{ND}(H^1))k\lambda\right]\Delta}\right.$$

$$-p_t^{t_u-\Delta}(\overline{H^1})e^{-\left[\text{num}(\mathcal{D}(H^1))\eta+\text{num}(\mathcal{ND}(H^1))\lambda\right]\Delta}+\sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}} p_t^{t_u-\Delta}(H^{1+v})k\left(1-e^{-k\lambda\Delta}\right)-\sum_{\substack{v\in\mathcal{D}(\overline{H}^1)\\ v\neq N}} p_t^{t_u-\Delta}(\overline{H}^{1+v})\left(1-e^{-\lambda\Delta}\right)$$

$$\left.+\sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}} p_t^{t_u-\Delta}(H^{1-v})k\left(1-e^{-\eta\Delta}\right)-\sum_{\substack{v\in\mathcal{ND}(\overline{H}^1)\\ v\neq 1}} p_t^{t_u-\Delta}(\overline{H}^{1-v})\left(1-e^{-\eta\Delta}\right)\right]\lambda\Delta \tag{82}$$

$$=\underbrace{p_t^{t_u-2\Delta}(H^{1+1})\left(1-e^{-\lambda\Delta}\right)\left[ke^{-\left[\text{num}(\mathcal{D}(H^1))\eta+\text{num}(\mathcal{ND}(H^1))k\lambda\right]\Delta}-e^{-\left[\text{num}(\mathcal{D}(H^1))\eta+\text{num}(\mathcal{ND}(H^1))\lambda\right]\Delta}\right]\lambda\Delta}_{1}$$

$$+\underbrace{p_t^{t_u-2\Delta}(H^{1-N})\left(1-e^{-\eta\Delta}\right)\left[ke^{-\left[\text{num}(\mathcal{D}(H^1))\eta+\text{num}(\mathcal{ND}(H^1))k\lambda\right]\Delta}-e^{-\left[\text{num}(\mathcal{D}(H^1))\eta+\text{num}(\mathcal{ND}(H^1))\lambda\right]\Delta}\right]\lambda\Delta}_{2}$$

$$+\underbrace{\sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}} p_t^{t_u-2\Delta}(H^{1+v+1})\left(1-e^{-\lambda\Delta}\right)\left(1-e^{-k\lambda\Delta}\right)k\lambda\Delta-\sum_{\substack{v\in\mathcal{D}(\overline{H}^1)\\ v\neq N}} p_t^{t_u-2\Delta}(H^{1+v+1})\left(1-e^{-\lambda\Delta}\right)\left(1-e^{-\lambda\Delta}\right)\lambda\Delta}_{3}$$

$$+\underbrace{\sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}} p_t^{t_u-2\Delta}(H^{1+v-N})\left(1-e^{-\eta\Delta}\right)\left(1-e^{-k\lambda\Delta}\right)k\lambda\Delta-\sum_{\substack{v\in\mathcal{D}(\overline{H}^1)\\ v\neq N}} p_t^{t_u-2\Delta}(H^{1+v-N})\left(1-e^{-\eta\Delta}\right)\left(1-e^{-\lambda\Delta}\right)\lambda\Delta}_{4}$$

$$+\underbrace{\sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}} p_t^{t_u-2\Delta}(H^{1-v+1})\left(1-e^{-\lambda\Delta}\right)\left(1-e^{-\eta\Delta}\right)k\lambda\Delta-\sum_{\substack{v\in\mathcal{ND}(\overline{H}^1)\\ v\neq 1}} p_t^{t_u-2\Delta}(H^{1-v+1})\left(1-e^{-\lambda\Delta}\right)\left(1-e^{-\eta\Delta}\right)\lambda\Delta}_{5}$$

$$+\underbrace{\sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}} p_t^{t_u-2\Delta}(H^{1-v-N})\left(1-e^{-\eta\Delta}\right)\left(1-e^{-\eta\Delta}\right)k\lambda\Delta-\sum_{\substack{v\in\mathcal{ND}(\overline{H}^1)\\ v\neq 1}} p_t^{t_u-2\Delta}(H^{1-v-N})\left(1-e^{-\eta\Delta}\right)\left(1-e^{-\eta\Delta}\right)\lambda\Delta}_{6}$$

$$+ p_t^{t_u-2\Delta}(H^1) k e^{-\left[2\mathrm{num}(\mathcal{D}(H^1))\eta + 2\mathrm{num}(\mathcal{ND}(H^1))k\lambda\right]\Delta}\lambda\Delta - p_t^{t_u-2\Delta}(\overline{H}^1) e^{-\left[2\mathrm{num}(\mathcal{D}(H^1))\eta + 2\mathrm{num}(\mathcal{ND}(H^1))\lambda\right]\Delta}\lambda\Delta$$
$$+ \sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}} p_t^{t_u-2\Delta}(H^{1+v}) k \left[e^{-\left[\mathrm{num}(\mathcal{D}(H^1))\eta + \mathrm{num}(\mathcal{ND}(H^1))k\lambda\right]\Delta} + e^{-\left[\mathrm{num}(\mathcal{D}(H^{1+v}))\eta + \mathrm{num}(\mathcal{ND}(H^{1+v}))k\lambda\right]\Delta}\right]\left(1 - e^{-k\lambda\Delta}\right)\lambda\Delta$$
$$- \sum_{\substack{v\in\mathcal{D}(\overline{H}^1)\\ v\neq N}} p_t^{t_u-2\Delta}(\overline{H}^{1+v}) \left[e^{-\left[\mathrm{num}(\mathcal{D}(H^1))\eta + \mathrm{num}(\mathcal{ND}(H^1))\lambda\right]\Delta} + e^{-\left[\mathrm{num}(\mathcal{D}(H^{1+v}))\eta + \mathrm{num}(\mathcal{ND}(H^{1+v}))\lambda\right]\Delta}\right]\left(1 - e^{-\lambda\Delta}\right)\lambda\Delta$$
$$+ \sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}} p_t^{t_u-2\Delta}(H^{1-v}) k \left[e^{-\left[\mathrm{num}(\mathcal{D}(H^1))\eta + \mathrm{num}(\mathcal{ND}(H^1))k\lambda\right]\Delta} + e^{-\left[\mathrm{num}(\mathcal{D}(H^{1-v}))\eta + \mathrm{num}(\mathcal{ND}(H^{1-v}))k\lambda\right]\Delta}\right]\left(1 - e^{-\eta\Delta}\right)\lambda\Delta$$
$$- \sum_{\substack{v\in\mathcal{ND}(\overline{H}^1)\\ v\neq 1}} p_t^{t_u-2\Delta}(\overline{H}^{1-v}) \left[e^{-\left[\mathrm{num}(\mathcal{D}(H^1))\eta + \mathrm{num}(\mathcal{ND}(H^1))\lambda\right]\Delta} + e^{-\left[\mathrm{num}(\mathcal{D}(H^{1-v}))\eta + \mathrm{num}(\mathcal{ND}(H^{1-v}))\lambda\right]\Delta}\right]\left(1 - e^{-\eta\Delta}\right)\lambda\Delta$$
$$+ \sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}} \sum_{\substack{v_2\in\mathcal{D}(H^{1+v})\\ v_2\neq 1}} p_t^{t_u-2\Delta}(H^{1+v+v_2}) k \left(1 - e^{-k\lambda\Delta}\right)\left(1 - e^{-k\lambda\Delta}\right)\lambda\Delta$$
$$- \sum_{\substack{v\in\mathcal{D}(\overline{H}^1)\\ v\neq N}} \sum_{\substack{v_2\in\mathcal{D}(\overline{H}^{1+v})\\ v_2\neq N}} p_t^{t_u-2\Delta}(\overline{H}^{1+v+v_2}) \left(1 - e^{-\lambda\Delta}\right)\left(1 - e^{-\lambda\Delta}\right)\lambda\Delta \quad (83)$$

$$+ \sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}} \sum_{\substack{v_2\in\mathcal{ND}(H^{1+v})\\ v_2\neq N}} p_t^{t_u-2\Delta}(H^{1+v-v_2}) k \left(1 - e^{-k\lambda\Delta}\right)\left(1 - e^{-\eta\Delta}\right)\lambda\Delta$$
$$- \sum_{\substack{v\in\mathcal{D}(\overline{H}^1)\\ v\neq N}} \sum_{\substack{v_2\in\mathcal{ND}(\overline{H}^{1+v})\\ v_2\neq 1}} p_t^{t_u-2\Delta}(\overline{H}^{1+v-v_2}) \left(1 - e^{-\lambda\Delta}\right)\left(1 - e^{-\eta\Delta}\right)\lambda\Delta$$
$$+ \sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}} \sum_{\substack{v_2\in\mathcal{D}(H^{1-v})\\ v_2\neq 1}} p_t^{t_u-2\Delta}(H^{1-v+v_2}) k \left(1 - e^{-k\lambda\Delta}\right)\left(1 - e^{-\eta\Delta}\right)\lambda\Delta$$
$$- \sum_{\substack{v\in\mathcal{ND}(\overline{H}^1)\\ v\neq 1}} \sum_{\substack{v_2\in\mathcal{D}(\overline{H}^{1-v})\\ v_2\neq N}} p_t^{t_u-2\Delta}(\overline{H}^{1-v+v_2}) \left(1 - e^{-\lambda\Delta}\right)\left(1 - e^{-\eta\Delta}\right)\lambda\Delta$$
$$+ \sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}} \sum_{\substack{v_2\in\mathcal{ND}(H^{1-v})\\ v_2\neq N}} p_t^{t_u-2\Delta}(H^{1-v-v_2}) k \left(1 - e^{-\eta\Delta}\right)\left(1 - e^{-\eta\Delta}\right)\lambda\Delta$$
$$- \sum_{\substack{v\in\mathcal{ND}(\overline{H}^1)\\ v\neq 1}} \sum_{\substack{v_2\in\mathcal{ND}(\overline{H}^{1-v})\\ v_2\neq 1}} p_t^{t_u-2\Delta}(\overline{H}^{1-v-v_2}) \left(1 - e^{-\eta\Delta}\right)\left(1 - e^{-\eta\Delta}\right)\lambda\Delta \quad (84)$$

The last equality in (84) follows by applying the Chapman-Kolmogorov equations to express $p_t^{t_u-\Delta}(\cdot)$ in terms of one-step backward transition probabilities $p_t^{t_u-2\Delta}(\cdot)$, and identifying all states from which next-period states can be reached, by the fact that in small time Δ at most one recovery or distress event can occur. Terms 1 and 2 in (84) are nonnegative because of Property 2. Terms 3, 4, 5, and 6 are nonnegative because $k > 1$. Therefore

$$\left[k p_t^{t_u}(H^1) - p_t^{t_u}(\overline{H^1})\right]\lambda\Delta \geq \text{last RHS in (84) excluding terms 1-6.} \quad (85)$$

Note that terms 1-6 in (84) derive from the fact that coupled states H^1 and $\overline{H}^1$ – or the states reached after a common sequence of distress and recoveries – have all elements in common except firm 1 and N, therefore in a time interval Δ can be reached from the same state, where either both 1 and N are in distress or both are not. By Property 2, and the fact that $k > 1$, these terms are nonnegative. Applying the Chapman-Kolmogorov equations to the RHS of (85) to condition on states at time $t_u - 3\Delta$, the resulting expression is then greater or equal than the same quantity that doesn't involve these terms. Iterating the procedure of backward induction until time $t_u - m\Delta$ and majorating the expression that neglects terms of the form 1-6 in (84), we can write:

$$
\begin{aligned}
&\left[k p_t^{t_u}(H^1) - p_t^{t_u}(\overline{H^1})\right]\lambda\Delta \geq \Big[p_t^{t_u-m\Delta}(H^1)k\mathcal{T}^0(m,H^1) - p_t^{t_u-m\Delta}(\overline{H}^1)\mathcal{T}^0(m,\overline{H}^1) \\
&+\sum_{w_1=+1,-1}\sum_{\substack{v_1\in\mathcal{S}^{w_1}(H^1)\\ v_1\neq l(w_1)}} p_t^{t_u-m\Delta}(H^{1+w_1v_1})k\mathcal{T}^1_{w_1}(m,H^1,v_1) - \sum_{w_1=+1,-1}\sum_{\substack{v_1\in\mathcal{S}^{w_1}(\overline{H}^1)\\ v_1\neq \bar{l}(w_1)}} p_t^{t_u-m\Delta}(\overline{H}^{1+w_1v_1})\mathcal{T}^1_{w_1}(m,\overline{H}^1,v_1) \\
&\quad+\sum_{w_1=+1,-1}\sum_{\substack{v_1\in\mathcal{S}^{w_1}(H^1)\\ v_1\neq l(w_1)}}\sum_{w_2=+1,-1}\sum_{\substack{v_2\in\mathcal{S}^{w_2}(H^{1+w_1v_1})\\ v_2\neq l(w_2)}} p_t^{t_u-m\Delta}(H^{1+w_1v_1+w_2v_2})k\mathcal{T}^2_{w_1,w_2}(m,H^1,v_1,v_2) \\
&\quad-\sum_{w_1=+1,-1}\sum_{\substack{v_1\in\mathcal{S}^{w_1}(\overline{H}^1)\\ v_1\neq \bar{l}(w_1)}}\sum_{w_2=+1,-1}\sum_{\substack{v_2\in\mathcal{S}^{w_2}(\overline{H}^{1+w_1v_1})\\ v_2\neq \bar{l}(w_2)}} p_t^{t_u-m\Delta}(\overline{H}^{1+w_1v_1+w_2v_2})\mathcal{T}^2_{w_1,w_2}(m,\overline{H}^1,v_1,v_2)
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{w_1=+1,-1}\sum_{\substack{v_1\in\mathcal{S}^{w_1}(H^1)\\ v_1\neq l(w_1)}}\sum_{w_2=+1,-1}\sum_{\substack{v_2\in\mathcal{S}^{w_2}(H^{1+w_1v_1})\\ v_2\neq l(w_2)}}\sum_{w_3=+1,-1}\sum_{\substack{v_3\in\mathcal{S}^{w_3}(H^{1+w_1v_1+w_2v_2})\\ v_3\neq l(w_3)}} p_t^{t_u-m\Delta}(H^{1+w_1v_1+w_2v_2+w_3v_3})k\times \\
&\qquad\times\mathcal{T}^3_{w_1,w_2,w_3}(m,H^1,v_1,v_2,v_3) \\
&-\sum_{w_1=+1,-1}\sum_{\substack{v_1\in\mathcal{S}^{w_1}(\overline{H}^1)\\ v_1\neq \bar{l}(w_1)}}\sum_{w_2=+1,-1}\sum_{\substack{v_2\in\mathcal{S}^{w_2}(\overline{H}^{1+w_1v_1})\\ v_2\neq \bar{l}(w_2)}}\sum_{w_3=+1,-1}\sum_{\substack{v_3\in\mathcal{S}^{w_3}(\overline{H}^{1+w_1v_1+w_2v_2})\\ v_3\neq \bar{l}(w_3)}} p_t^{t_u-m\Delta}(\overline{H}^{1+w_1v_1+w_2v_2+w_3v_3})\times \\
&\qquad\times\mathcal{T}^3_{w_1,w_2,w_3}(m,\overline{H}^1,v_1,v_2,v_3) \\
&\qquad\qquad\cdots\cdots \\
&\qquad\qquad\cdots\cdots \\
&+\sum_{w_1=+1,-1}\sum_{\substack{v_1\in\mathcal{S}^{w_1}(H^1)\\ v_1\neq l(w_1)}}\sum_{w_2=+1,-1}\sum_{\substack{v_2\in\mathcal{S}^{w_2}(H^{1+w_1v_1})\\ v_2\neq l(w_2)}}\cdots\sum_{w_m=+1,-1}\sum_{\substack{v_m\in\mathcal{S}^{w_m}(H^{1+\sum_{h=1}^{m-1}w_hv_h})\\ v_m\neq l(w_m)}} p_t^{t_u-m\Delta}(H^{1+\sum_{h=1}^{m}w_hv_h})k\times \\
&\qquad\times\mathcal{T}^m_{w_1,\ldots,w_m}(m,H^1,v_1,\ldots,v_m) \\
&-\sum_{w_1=+1,-1}\sum_{\substack{v_1\in\mathcal{S}^{w_1}(\overline{H}^1)\\ v_1\neq \bar{l}(w_1)}}\sum_{w_2=+1,-1}\sum_{\substack{v_2\in\mathcal{S}^{w_2}(\overline{H}^{1+w_1v_1})\\ v_2\neq \bar{l}(w_2)}}\cdots\sum_{w_m=+1,-1}\sum_{\substack{v_m\in\mathcal{S}^{w_m}(\overline{H}^{1+\sum_{h=1}^{m-1}w_hv_h})\\ v_m\neq \bar{l}(w_m)}} p_t^{t_u-m\Delta}(\overline{H}^{1+\sum_{h=1}^{m}w_hv_h})\times \\
&\qquad\times\mathcal{T}^m_{w_1,\ldots,w_m}(m,\overline{H}^1,v_1,\ldots,v_m)\Big]\lambda\Delta, \quad (86)
\end{aligned}
$$

where

$$
\mathcal{T}^0(m,H) = \begin{cases} 1 \text{ if } m=0 \\ \mathcal{T}^0(m-1,H)e^{-\left[\text{num}(\mathcal{D}(H))\eta+\text{num}(\mathcal{ND}(H))(k\lambda\mathbf{1}(H\equiv H^1)+\lambda\mathbf{1}(H\equiv\overline{H}^1))\lambda\right]\Delta} & \text{otherwise}\end{cases} \quad (87)
$$

$$
\mathcal{T}^1_{w_1}(m,H,v_1) = \begin{cases} 0 \text{ if } m=0 \\ \mathcal{T}^0(m-1,H)\left(1-e^{-\tilde{\lambda}(w_1,H)\Delta}\right)+\mathcal{T}^1_{w_1}(m-1,H,v_1)\times \\ \times e^{-\left[\text{num}(\mathcal{D}(H^{+w_1v_1}))\eta+\text{num}(\mathcal{ND}(H^{+w_1v_1}))(k\lambda\mathbf{1}(H\equiv H^1)+\lambda\mathbf{1}(H\equiv\overline{H}^1))\right]\Delta} & \text{otherwise}\end{cases} \quad (88)
$$

$$\mathcal{T}^2_{w_1,w_2}(m,H,v_1,v_2) = \begin{cases} 0 \text{ if } m=0 \\ \mathcal{T}^1_{w_1}(m-1,H,v_1)\left(1-e^{-\tilde{\lambda}(w_2,H)\Delta}\right)+\mathcal{T}^2_{w_1,w_2}(m-1,H,v_1,v_2)\times \\ \times e^{-\left[\text{num}(\mathcal{D}(H^{+w_1v_1+w_2v_2}))\eta+\text{num}(\mathcal{ND}(H^{+w_1v_1+w_2v_2}))(k\lambda\mathbf{1}(H\equiv H^1)+\lambda\mathbf{1}(H\equiv\overline{H}^1))\right]\Delta} & \text{otherwise} \end{cases} \tag{89}$$

$$\ldots \tag{90}$$

$$\mathcal{T}^m_{w_1,\ldots,w_m}(m,H,v_1,\ldots,v_m) = \begin{cases} 0 \text{ if } m=0 \\ \mathcal{T}^{m-1}_{w_1,\ldots,w_{m-1}}(m-1,H,v_1,\ldots,v_{m-1})\left(1-e^{-\tilde{\lambda}(w_m,H)\Delta}\right) \\ +\mathcal{T}^m_{w_1,\ldots,w_m}(m-1,H,v_1,\ldots,v_m)e^{-\text{num}(\mathcal{D}(H^{+\sum_{h=1}^m w_hv_h}))\eta\Delta}\times \\ \times e^{-\text{num}(\mathcal{ND}(H^{+\sum_{h=1}^m w_hv_h}))(k\lambda\mathbf{1}(H\equiv H^1)+\lambda\mathbf{1}(H\equiv\overline{H}^1))\Delta} & \text{otherwise} \end{cases} \tag{91}$$

$\mathbf{1}(\,\cdot\,)$ denotes the indicator function of an event. Furthermore:

$$\mathcal{S}^w(H) = \begin{cases} \mathcal{D}(H) & \text{if } w=+1 \\ \mathcal{ND}(H) & \text{if } w=-1 \end{cases} \tag{92}$$

$$l(w) = \begin{cases} 1 & \text{if } w=+1 \\ N & \text{if } w=-1 \end{cases} \tag{93}$$

$$\bar{l}(w) = \begin{cases} N & \text{if } w=+1 \\ 1 & \text{if } w=-1 \end{cases} \tag{94}$$

$$\tilde{\lambda}(w,H) = \begin{cases} k\lambda\mathbf{1}(H\equiv H^1)+\lambda\mathbf{1}(H\equiv\overline{H}^1) & \text{if } w=+1 \\ \eta & \text{if } w=-1 \end{cases} \tag{95}$$

For $m=n^\Delta$, we have $t_u-m\Delta=t$, so that

$$p_t^{t_u-n^\Delta\Delta}(H) = \begin{cases} 1 \text{ if } \mathbf{H}_t\equiv H \\ 0 \text{ otherwise} \end{cases}, \quad \forall H \tag{96}$$

By assumption, in the initial state $\mathbf{H}_t$ both firms 1 and N are not in distress, while firm 1 is in distress in any state $H^{1+\sum_{h=1}^s w_hv_h}$, and firm N is in distress in any state $\overline{H}^{1+\sum_{h=1}^s w_hv_h}$, $s=1,\ldots,n^\Delta$. This implies that taking $m=n^\Delta$ in expression (86), its RHS vanishes, and claim (71) follows.

To see that claim (72) holds, notice that

$$\lim_{k\to\infty} p_t^{t_u}(H^1)=0, \tag{97}$$

because, according to (75)-(76),

$$\lim_{k\to\infty} Prob\left(\mathbf{H}_{t_u}=H^1\middle|\,\mathbf{H}_{t_u-\Delta}=H^1\right)=0 \tag{98}$$

$$\lim_{k\to\infty} Prob\left(\mathbf{H}_{t_u}=H^{1-j}\middle|\,\mathbf{H}_{t_u-\Delta}=H^1\right)=1 \tag{99}$$

In other words, as the propagation of distress approaches immediacy, the probability of a future state where firm 1 is in distress and some other firm is not approaches 0. (97) does not hold instead for states of the form $\overline{H^1}$, so that

$$\lim_{k\to\infty} p_t^{t_u}(H^1)-p_t^{t_u}(\overline{H^1})<0. \tag{100}$$

Then there must exist a $k^*(\lambda,\eta,k_h,k_l)$, hence also dependent on n, such that $p_t^{t_u}(H^1)<p_t^{t_u}(\overline{H^1})$ for $k>k^*(\lambda,\eta,k_h,k_l)$. We haven't been able to show that $p_t^{t_u}(H^1)-p_t^{t_u}(\overline{H^1})$ is monotonically decreasing in k for $k>1$. We provide some supportive numerical evidence, on a finite economy with $n=10$ firms. Table X reports the critical

$k^*(\lambda,\eta,k_h,k_l)$ for different values of λ and η, and the percentage of violations of the condition $p_t^{t_u}(H^1) - p_t^{t_u}(\overline{H^1})$ for $k < k^*(\lambda,\eta,k_h,k_l)$, for all paired states $(H^1,\overline{H^1})$, and initial states $\mathbf{H}_t$. In all cases the percentages approach zero monotonically as $k \to k^*(\lambda,\eta,k_h,k_l)$.

Insert Table X

To see that claim (74) holds, we write:

$$p_t^{t_u}(H^1)\left[\mathcal{A}^N(H^1) - \mathcal{A}^1(H^1)\right] - p_t^{t_u}(\overline{H}^1)\left[\mathcal{A}^1(\overline{H}^1) - \mathcal{A}^N(\overline{H}^1)\right] = p_t^{t_u}(H^1)\left[\sum_{\substack{j\in\mathcal{ND}(H^1)\\ j\neq N}} k\lambda\Delta\left[C^N(H^{1-j}) - C^1(H^{1-j})\right.\right.$$
$$\left. - \left(C^N(H^1) - C^1(H^1)\right)\right] + \sum_{\substack{j\in\mathcal{D}(H^1)\\ j\neq 1}} \eta\Delta\left[\left(C^N(H^{1+j}) - C^1(H^{1+j})\right) - \left(C^N(H^1) - C^1(H^1)\right)\right]$$
$$\left. +(1-a)(C^N(H^1) - C^1(H^1)) - k\lambda\Delta\left(C^N(H^1) - C^1(H^1)\right) - \eta\Delta\left(C^N(H^1) - C^1(H^1)\right)\right] \quad (101)$$

$$- p_t^{t_u}(\overline{H}^1)\left[\sum_{\substack{j\in\mathcal{ND}(\overline{H}^1)\\ j\neq N}} \lambda\Delta\left[C^N(\overline{H}^{1-j}) - C^1(\overline{H}^{1-j})\right.\right.$$
$$\left. - \left(C^N(\overline{H}^1) - C^1(\overline{H}^1)\right)\right] + \sum_{\substack{j\in\mathcal{D}(\overline{H}^1)\\ j\neq 1}} \eta\Delta\left[C^N(\overline{H}^{1+j}) - C^1(\overline{H}^{1+j}) - \left(C^N(\overline{H}^1) - C^1(\overline{H}^1)\right)\right]$$
$$\left. +(1-a)(C^N(\overline{H}^1) - C^1(\overline{H}^1)) - \lambda\Delta\left(C^N(\overline{H}^1) - C^1(\overline{H}^1)\right) - \eta\Delta\left(C^N(\overline{H}^1) - C^1(\overline{H}^1)\right)\right] \quad (102)$$

Using the homogeneous dividends assumption *iii*), we have $C^N(H^1) - C^1(H^1) = C^1(\overline{H}^1) - C^N(\overline{H}^1)$, $C^N(H^{1-j}) - C^1(H^{1-j}) = C^1(\overline{H}^{1-j}) - C^N(\overline{H}^{1-j})$ if $j \neq 1, N$, $C^N(H^{1+j}) - C^1(H^{1+j}) = C^1(\overline{H}^{1+j}) - C^N(\overline{H}^{1+j})$, if $j \neq 1, N$. Moreover $\mathcal{ND}(H^1)$ excluding N coincides with $\mathcal{ND}(\overline{H}^1)$ excluding 1, and $\mathcal{D}(H^1)$ excluding 1 coincides with $\mathcal{D}(\overline{H}^1)$ excluding N. These facts allow to collect terms in (102) and obtain (74). □

Lemma 5. *The condition $k^*(\lambda,\eta,k_h,k_l) < k < k^{**}$, where k^{**} solves*

$$1 - K^\lambda(k+1)\Delta - K^\eta\Delta = 0 \quad (103)$$

is sufficient for

$$\lim_{n\to\infty} \mathcal{R}^N - \mathcal{R}^i \geq 0 \quad (104)$$

to hold.

By virtue of (67) and (68):

$$\begin{aligned}
\mathcal{R}^N - \mathcal{R}^1 &= \sum_{j=1}^{n} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\left[\mathbf{\bar{1}}'(\mathbf{H}_t^{\pm j})\sum_{u=0}^{\infty}\exp(-(\mathbf{a}+\mathbf{A}^H)(t_u-t))\left(\mathcal{A}^N - \mathcal{A}^1\right)\right] \\
&= \left(\mathcal{R}^N_{ND} - \mathcal{R}^1_{ND}\right) + \lambda^1(\mathbf{H}_t)\theta^1(\mathbf{H}_t)\sum_{u=0}^{\infty} e^{-a(t_u-t)}\left[\sum_{H^1} Prob\left(H_{t_u} = H^1 \middle| \mathbf{H}_t^{-1}\right)\left(\mathcal{A}^N(H^1) - \mathcal{A}^1(H^1)\right)\right. \\
&\quad \left. + \sum_{\overline{H}^1} Prob\left(H_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{-1}\right)\left(\mathcal{A}^N(\overline{H}^1) - \mathcal{A}^1(\overline{H}^1)\right)\right]
\end{aligned}$$

$$+\lambda^N(\mathbf{H}_t)\theta^N(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-a(t_u-t)}\left[\sum_{H^1}Prob\left(H_{t_u}=H^1\middle|\,\mathbf{H}_t^{-N}\right)\left(\mathcal{A}^N(H^1)-\mathcal{A}^1(H^1)\right)\right.$$

$$\left.+\sum_{\overline{H}^1}Prob\left(H_{t_u}=\overline{H}^1\middle|\,\mathbf{H}_t^{-N}\right)\left(\mathcal{A}^N(\overline{H}^1)-\mathcal{A}^1(\overline{H}^1)\right)\right] \tag{105}$$

with

$$\mathcal{R}_{ND}^N-\mathcal{R}_{ND}^1=\sum_{\substack{j=1\\ j\neq 1,N}}^{n}\tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-a(t_u-t)}\left(\sum_{H^1}Prob\left(H_{t_u}=H^1\middle|\,\mathbf{H}_t^{\pm j}\right)\left(\mathcal{A}^N(H^1)-\mathcal{A}^1(H^1)\right)\right.$$

$$\left.+\sum_{\overline{H}^1}Prob\left(H_{t_u}=\overline{H}^1\middle|\,\mathbf{H}_t^{\pm j}\right)\left(\mathcal{A}^N(\overline{H}^1)-\mathcal{A}^1(\overline{H}^1)\right)\right) \tag{106}$$

We note that in the expression for $\mathcal{R}_{ND}^N-\mathcal{R}_{ND}^1$, in states $\mathbf{H}_t^{\pm j}$ firms 1 and N are necessarily both not in distress, because in the initial state $\mathbf{H}_t$ they are not by assumption. We also note that the only relevant states at time t_u in expression (105), are necessarily the paired states of the form H^1 and $\overline{H}^1$ of Lemma 4: any state of (non) distress for the economy excluding firms 1 and N gives rise to four states; two of them are paired states H^1 and $\overline{H}^1$, and in the remaining two firm 1 and N are both in distress or both not in distress. In the latter case expression $\mathcal{A}^N(H)-\mathcal{A}^1(H)$ vanishes, according to Lemma 3. Using (74) of Lemma 4, expression (106) reads explicitly:

$$\begin{aligned}
\mathcal{R}_{ND}^N-\mathcal{R}_{ND}^1 &= \sum_{\substack{j=1\\ j\neq 1,N}}^{n}\tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-a(t_u-t)}\sum_{H^1}\mathcal{U}(H^1,u)\\
\mathcal{U}(H^1,u) &= \Bigg\{\left[kProb\left(\mathbf{H}_{t_u}=H^1\middle|\,\mathbf{H}_t\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\,\mathbf{H}_t\right)\right]\lambda\Delta\times\\
&\left[\sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}}\left(C^N(H^{1-v})-C^1(H^{1-v})-\left(C^N(H^1)-C^1(H^1)\right)\right)-\left(C^N(H^1)-C^1(H^1)\right)\right]\\
&+\left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\,\mathbf{H}_t\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\,\mathbf{H}_t\right)\right]\Big[(1-a)(C^N(H^1)-C^1(H^1))+\\
&\sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}}\eta\Delta\left(\left(C^N(H^{1+v})-C^1(H^{1+v})\right)-\left(C^N(H^1)-C^1(H^1)\right)\right)-\eta\Delta\left(C^N(H^1)-C^1(H^1)\right)\Big]\Bigg\}
\end{aligned} \tag{107}$$

Letting $n\to\infty$, we distinguish three possible cases concerning a given state H^1:

1. $\lim_{n\to\infty}\mathrm{num}\left(\mathcal{ND}(H^1)\right)=\infty$, $\lim_{n\to\infty}\mathrm{num}\left(\mathcal{D}(H^1)\right)=K$, for some finite integer K.

   Setting:

$$K_n^1 = \sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}}\left(C^N(H^{1-v})-C^1(H^{1-v})-\left(C^N(H^1)-C^1(H^1)\right)\right), \tag{108}$$

   we have

$$\sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}}\left(C^N(H^{1-v})-C^1(H^{1-v})-\left(C^N(H^1)-C^1(H^1)\right)\right)-\left(C^N(H^1)-C^1(H^1)\right)=K^1(n)+o(K^1(n))\geq 0 \tag{109}$$

and

$$(1-a)(C^N(H^1)-C^1(H^1))+\sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}}\Big(\big(C^N(H^{1+v})-C^1(H^{1+v})\big)-\big(C^N(H^1)-C^1(H^1)\big)\Big)$$
$$-\big(C^N(H^1)-C^1(H^1)\big)=o(K^1(n)) \quad (110)$$

for n large. The sign of the RHSs in (109) derives from the fact that $C^N(H^{1-v})-C^1(H^{1-v})\geq C^N(H^1)-C^1(H^1)$ for $\gamma>1$. Due to (71) and (72) we can conclude that $\lim_{n\to\infty}\mathcal{U}(H^1,u)\geq 0$.

2. $\lim_{n\to\infty}\text{num}\left(\mathcal{D}(H^1)\right)=\infty$, $\lim_{n\to\infty}\text{num}\left(\mathcal{ND}(H^1)\right)=K$, for some finite integer K.

   Setting:

$$K_n^2 \;=\; \sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}}\left(C^N(H^1)-C^1(H^1)-\big(C^N(H^{1+v})-C^1(H^{1+v})\big)\right), \quad (111)$$

   we have

$$(1-a)(C^N(H^1)-C^1(H^1))+\sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}}\Big(\big(C^N(H^{1+v})-C^1(H^{1+v})\big)-\big(C^N(H^1)-C^1(H^1)\big)\Big)$$
$$-\big(C^N(H^1)-C^1(H^1)\big)=-[K^2(n)+o(K^2(n))]\leq 0 \quad (112)$$

   for n large, and

$$\sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}}\left(C^N(H^{1-v})-C^1(H^{1-v})-\big(C^N(H^1)-C^1(H^1)\big)\right)-\big(C^N(H^1)-C^1(H^1)\big)\;=\;o(K^2(n)) \quad (113)$$

   The sign of the RHSs in (112) derives from the fact that $C^N(H^{1+v})-C^1(H^{1+v})\leq C^N(H^1)-C^1(H^1)$ for $\gamma>1$. Since $K^2(n)$ is bounded $\forall n$ because of Assumption 1 and (62), claims (71) and (72) let us conclude that $\lim_{n\to\infty}\mathcal{U}(H^1,u)=0$.

3. $\lim_{n\to\infty}\text{num}\left(\mathcal{D}(H^1)\right)=\infty$, $\lim_{n\to\infty}\text{num}\left(\mathcal{ND}(H^1)\right)=\infty$. By the reasoning as above: $\lim_{n\to\infty}\mathcal{U}(H^1,u)\geq 0$.

We then have

$$\lim_{n\to\infty}\mathcal{R}^N_{ND}-\mathcal{R}^1_{ND}\geq 0 \quad (114)$$

It is clear from (105) and (114) that

$$\lim_{n\to\infty}\mathcal{R}^N-\mathcal{R}^1=\mathcal{R}^N_{ND}-\mathcal{R}^1_{ND}+o(\mathcal{R}^N_{ND}-\mathcal{R}^1_{ND}), \quad (115)$$

because, by the assumption that in the initial state $\mathbf{H}_t$ firm 1 and N are not in distress, there are only two states $\mathbf{H}_t^{\pm j}$ where firm 1 or firm N is in distress, regardless of n. □

Using (104) we obtain:

$$\lim_{n\to\infty}\left[\mu_t^N-\gamma\sigma^2-\sum_{j=1}^{n}\tilde{\lambda}^j\right]P^N(\mathbf{H}_t)=\lim_{n\to\infty}-\mathcal{R}^N\leq\lim_{n\to\infty}-\mathcal{R}^1=\lim_{n\to\infty}\left[\mu_t^1-\gamma\sigma^2-\sum_{j=1}^{n}\tilde{\lambda}^j\right]P^1(\mathbf{H}_t) \quad (116)$$

so that

$$\lim_{n\to\infty}\left[\mu_t^N-\gamma\sigma^2-\sum_{j=1}^n\tilde{\lambda}^j\right]\leq\left[\mu_t^1-\gamma\sigma^2-\sum_{j=1}^n\tilde{\lambda}^j\right]\frac{P^1(\mathbf{H}_t)}{P^N(\mathbf{H}_t)}\leq\lim_{n\to\infty}\left[\mu_t^1-\gamma\sigma^2-\sum_{j=1}^n\tilde{\lambda}_n^j\right] \tag{117}$$

The last inequality follows from the fact that $P^N(\mathbf{H}_t) > P^1(\mathbf{H}_t)$ for large n, which is a consequence of the reasoning above, once we notice that

$$P^N(\mathbf{H}_t)-P^1(\mathbf{H}_t)=\overline{\mathbf{1}}'(\mathbf{H}_t)\sum_{u=0}^{\infty}\exp(-(\mathbf{a}+\mathbf{A}^H)(t_u-t))\left(\mathcal{A}^N-\mathcal{A}^1\right), \tag{118}$$

and that firms 1 and N are not in distress in $\mathbf{H}_t$. We can conclude that $\mu_t^1\geq\mu_t^N$ for n large.
□

Proof of Proposition 4. We use the same notation of the proof of Proposition 3. In order to focus on network connectivity, we assume complete-information, so that the business cycle factor S is observable. For simplicity we drop the explicit dependence on S from intensity parameters. We can think of it as an additional argument of the aggregate state of distress (or not) $\mathbf{H}$. H will denote a generic realization of $\mathbf{H}$. Assumptions 1 and Assumption 2 of Proposition 3 are replaced by the following:

Assumption 3. *Dividends are deterministic functions of the economy size n, and asymptotically homogeneous:*

$$x_t^i(H)=\begin{cases}\overline{f}^i(n) & \text{if } H_t^i=0\\ \underline{f}^i(n) & \text{if } H_t^i=1\end{cases}\qquad i=1,\dots n \tag{119}$$

with $\lim_{n\to\infty}x_t^i(H)=\lim_{n\to\infty}x_t^j(H)$, $\forall i,j$. *Moreover*

$$\lim_{n\to\infty}\left(\frac{\sum_{j=1}^n x_t^j(H)}{\sum_{j=1}^n x_t^j(\mathbf{H}_t)}\right)^{-\gamma}\frac{x_t^i(H)}{x_t^i(\mathbf{H}_t)}=c(H,\mathbf{H}_t) \tag{120}$$

with $0<c^i(H,\mathbf{H}_t)<\infty$, *for all possible states H.*

Assumption 4. *For a given economy size n, intensities $\lambda^i(H)$ and $\eta^j(H)$, $j=1,\dots,n$ are independent conditionally on the state H, and they are realizations of common (across firms) distributions $F^\lambda(n,H)$ and $F^\eta(n,H)$. These distributions are such that the following condition holds*

$$\begin{array}{rcl}\lim_{n\to\infty}\sum_{v=1}^n\lambda^v(H) & = & K^\lambda(H)<\infty\\ \lim_{n\to\infty}\sum_{v=1}^n\eta^v(H) & = & K^\eta(H)<\infty\end{array} \tag{121}$$

which implies

$$\lim_{n\to\infty}\lambda^i(H)=\lim_{n\to\infty}\eta^i(H)=0\qquad\forall H,\ i=1,\dots,n \tag{122}$$

For simplicity we drop the dependence on n from the $\lambda^i(\cdot)$, $\eta(\cdot)$ and $x_t^i(\cdot)$.

Consider an initial state $\mathbf{H}_t$ and an economy size n. Following the lines of the proof of Proposition 3, we redefine $\mathcal{R}^i$ as

$$\begin{aligned}\mathcal{R}^i &= \sum_{j=1}^n\tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\left[\overline{\mathbf{1}}'(\mathbf{H}_t^{\pm j})\sum_{u=0}^{\infty}\exp(-(\mathbf{a}+\mathbf{A}^H)(t_u-t))\mathbf{A}\mathbf{c}^i\right]\\ &= \mathcal{R}_{ND}^i+\lambda^i(\mathbf{H}_t)\theta^i(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-a(t_u-t)}\left[\sum_H Prob\left(H_{t_u}=H|\,\mathbf{H}_t^{-i}\right)\mathcal{A}^i(H)\right]\end{aligned} \tag{123}$$

The reason to partition $\mathcal{R}^i$ in (123) is to isolate the only term where firm j is in distress in $\mathbf{H}_t^{\pm j}$. We have set

$$\mathcal{R}_{ND}^i = \sum_{\substack{j=1\\ j\neq i}}^{n} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t) \sum_{u=0}^{\infty} e^{-a(t_u-t)} \left(\sum_H Prob\left(H_{t_u} = H | \, \mathbf{H}_t^{\pm j} \right) \mathcal{A}^i(H) \right) \tag{124}$$

$\mathbf{c}^i = \mathbf{C}^i / x_t^i(\mathbf{H}_t)$ is the vector of dividends paid by firm i in each possible state H, discounted by the marginal rate of intertemporal substitution, and scaled by the current dividend $x_t^i(\mathbf{H}_t)$. We denote by $c^i(H)$ the entry of vector $\mathbf{c}^i$ corresponding to state H. We have also set $\mathcal{A}^i = \mathbf{A}\mathbf{c}^i$, with $\mathcal{A}^i(H)$ the entry of this vector corresponding to state H. We use the familiar representation for the risk premium of the $i-$th equity security:[22]

$$\left[\mu_t^i - \gamma\sigma^2 - \sum_{j=1}^{n} \tilde{\lambda}^j \right] \frac{P^i(\mathbf{H}_t)}{x_t^i(\mathbf{H}_t)} \quad = \quad -\mathcal{R}^i \tag{125}$$

In particular, we consider the limit $\lim_{n\to\infty} \mathcal{R}^{i_1} - \mathcal{R}^{i_2}$, for any pair of firms i_1 and i_2.

For convenience, the two symmetric networks of Figure 1, disconnected and fully connected, are considered first and last respectively, while the 'Star' network of Figure 2 is considered in between.

'Disconnected' Network of Figure 1a.

If firms are not connected, the distribution of firms' intensity parameters is independent of the state H, so that $\lambda^i(\mathbf{H}_t) = \lambda^i$ and $\eta^i(\mathbf{H}_t) = \eta^i$, $i = 1, \ldots, n$, are independent and with identical distributions $F^\lambda(n)$ and $F^\eta(n)$, respectively.

Lemma 3 holds in this context, therefore we need only consider paired states H^{i_1} and $\overline{H}^{i_1}$, having all firms' (distress or not) states in common, except for firms i_1 and i_2: the former is in distress in H^{i_1} but not in $\overline{H}^{i_1}$. The converse holds for i_2.

As in (107) of Proposition 3, taking into account that $k = 1$ and the asymptotic homogeneity of dividends in Assumption 4, we have, for n large:

$$\begin{aligned}
\mathcal{R}_{ND}^{i_2} - \mathcal{R}_{ND}^{i_1} \quad &= \quad \sum_{\substack{j=1\\ j\neq i_1,i_2}}^{n} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t) \sum_{u=0}^{\infty} e^{-a(t_u-t)} \sum_{H^{i_1}} \mathcal{U}(H^{i_1}, u) \\
\mathcal{U}(H^{i_1}, u) \quad &= \quad \left[Prob\left(\mathbf{H}_{t_u} = H^{i_1} | \, \mathbf{H}_t^{\pm j} \right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^{i_1} \middle| \, \mathbf{H}_t^{\pm j} \right) \right] \mathcal{B}^1(H^{i_1}) \\
&\quad + \left[Prob\left(\mathbf{H}_{t_u} = H^{i_1} | \, \mathbf{H}_t^{\pm j} \right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^{i_1} \middle| \, \mathbf{H}_t^{\pm j} \right) \right] \mathcal{B}^2(H^{i_1}) \\
\mathcal{B}^1(H^{i_1}) \quad &= \quad \left[\underbrace{\sum_{\substack{v\in\mathcal{ND}(H^{i_1})\\ v\neq i_2}} \lambda^v \Delta \left(c^{i_2}(H^{i_1-v}) - c^{i_1}(H^{i_1-v}) - \left(c^{i_2}(H^{i_1}) - c^{i_1}(H^{i_1}) \right) \right) -}_{1} \right. \\
&\quad \left. \underbrace{\left(\lambda^{i_2} - \lambda^{i_1} \right) \left(c^{i_2}(H^{i_1}) - c^{i_1}(H^{i_1}) \right) \Delta}_{2} \right]
\end{aligned} \tag{126}$$

[22]Remind that $\tilde{\lambda}^i = H_t^i \eta^i + (1 - H_t^i)\lambda^i$

$$\mathcal{B}^2(H^{i_1}) = \underbrace{(1-a)(c^{i_2}(H^{i_1})-c^{i_1}(H^{i_1}))-\Delta\left(\eta^{i_1}c^{i_2}(H^{i_1})-\eta^{i_2}c^{i_1}(H^{i_1})\right)}_{3}$$
$$\underbrace{+\sum_{\substack{v\in\mathcal{D}(H^{i_1})\\ v\neq i_1}}\eta^v\Delta\left[\left(c^{i_2}(H^{i_1+v})-c^{i_1}(H^{i_1+v})\right)-\left(c^{i_2}(H^{i_1})-c^{i_1}(H^{i_1})\right)\right]}_{4}$$

As $n\to\infty$, given a generic H^{i_1}, we have either $\lim_{n\to\infty}\mathcal{ND}(H^{i_1})=K$, for some integer $K<\infty$, or $\lim_{n\to\infty}\mathcal{ND}(H^{i_1})=\infty$. In the former case, $\lim_{n\to\infty}\mathcal{B}^1(H^{i_1})=0$ because of Assumption 1 and (122). In the latter, $\mathcal{B}^1(H^{i_1})$ is an infinite sum of independent random variables, because of Assumption 1. We assume that F^λ and $(\overline{f}(n),\underline{f}(n))$ are such that the Lindberg condition – see Durret (1995) – is satisfied, which is not restrictive in light of (122) and (60). The Lindberg-Feller theorem then mandates that $\lim_{n\to\infty}\mathcal{B}^1(H^{i_1})=\epsilon_1$, where $\epsilon_1\sim N(\mu_1,\sigma_1)$.[23] Similarly, we either have $\mathcal{B}^2(H^{i_1})\approx 0$ or $\mathcal{B}^2(H^{i_1})\approx\epsilon_2\sim N(\mu_2,\sigma_2)$ for n large. We now show that

$$\lim_{n\to\infty} Prob\left(\mathbf{H}_{t_u}=H^{i_1}\Big|\mathbf{H}_t^{\pm j}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^{i_1}\Big|\mathbf{H}_t^{\pm j}\right)=0 \tag{127}$$

We refer to the proof of Lemma 4 above, where we set $k=1$, because the network is disconnected, as assign its own λ^j (η^j) to the distress (recovery) event of firm j, instead of having homogeneous parameters. Terms 1 and 2 in (81) become:

$$\underbrace{p_t^{t_u-\Delta}(H^{i_1+i_1})\left(1-e^{-\lambda^{i_1}\Delta}\right)-p_t^{t_u-\Delta}(H^{i_1+i_1})\left(1-e^{-\lambda^{i_2}\Delta}\right)}_{1} \tag{128}$$
$$\underbrace{p_t^{t_u-\Delta}(H^{i_1-i_2})\left(1-e^{-\eta^{i_2}\Delta}\right)-p_t^{t_u-\Delta}(H^{i_1-i_2})\left(1-e^{-\eta^{i_1}\Delta}\right)}_{2}$$

For n large, (122) implies $\lambda^{i_2}\approx\lambda^{i_1}\approx\eta^{i_2}\approx\eta^{i_1}\approx 0$, so that term $1\approx$ term $2\approx 0$. Terms 1 and 2 of (84) become

$$p_t^{t_u-2\Delta}(H^{i_1+i_1})\left[\left(1-e^{-\lambda^{i_1}\Delta}\right)e^{-\left[\sum_{v\in\mathcal{D}(H^1)}\eta^v+\sum_{v\in\mathcal{ND}(H^1)}\lambda^v\right]\Delta}\right. \tag{129}$$
$$\left.-\left(1-e^{-\lambda^{i_2}\Delta}\right)e^{-\left[\sum_{v\in\mathcal{D}(H^1)}\eta^v+\sum_{v\in\mathcal{ND}(H^1)}\lambda^v\right]\Delta}\right]\lambda\Delta \quad (\text{term } 1) \tag{130}$$
$$+p_t^{t_u-2\Delta}(H^{i_1-i_2})\left[\left(1-e^{-\eta^{i_2}\Delta}\right)e^{-\left[\sum_{v\in\mathcal{D}(H^1)}\eta^v+\sum_{v\in\mathcal{ND}(H^1)}\lambda^v\right]\Delta}\right. \tag{131}$$
$$\left.-\left(1-e^{-\eta^{i_1}\Delta}\right)e^{-\left[\sum_{v\in\mathcal{D}(H^1)}\eta^v+\sum_{v\in\mathcal{ND}(H^1)}\lambda^v\right]\Delta}\right]\lambda\Delta \quad (\text{term } 2) \tag{132}$$

As $n\to\infty$, the summations at the exponentials in square brackets converge to the same limit. Term 3 in (84) becomes

$$\lim_{n\to\infty}\left[\left(1-e^{-\lambda^{i_1}\Delta}\right)-\left(1-e^{-\lambda^{i_2}\Delta}\right)\right]\sum_{\substack{v\in\mathcal{D}(H^{i_1})\\ v\neq 1}}p_t^{t_u-2\Delta}(H^{i_1+v+i_1})\left(1-e^{-\lambda^v\Delta}\right)=0, \tag{133}$$

because $\lambda^{i_1}\approx\lambda^{i_2}$ and Assumption 4 guarantees that the summation converges to a bounded limit. The same reasoning applies to terms 4-6 in expression (84), and to terms of this type that arise from further backward substitutions (see the proof of Lemma 4). The rest of the proof is unchanged. Since $p_t^t(H)=0$ for any H of the type H^{i_1} and $\overline{H}^{i_1}$, by the assumption that i_1 and i_2 are not in distress in $\mathbf{H}_t^{\pm j}$, the limit (127) follows. In light of

[23]Mean and variance parameters do not play a specific role, hence we leave them unspecified.

(126) we have

$$\lim_{n\to\infty} \mathcal{R}^{i_2}_{ND} - \mathcal{R}^{i_1}_{ND} = 0, \tag{134}$$

so that

$$\begin{aligned}\lim_{n\to\infty} \mathcal{R}^{i_2} - \mathcal{R}^{i_1} \quad = \quad & \sum_{j=i_1,i_2} \lambda^j \theta^j(\mathbf{H}_t) \sum_{u=0}^{\infty} e^{-a(t_u-t)} \sum_{H^{i_1}} \Big\{ \Big[Prob\left(\mathbf{H}_{t_u} = H^{i_1} \middle| \mathbf{H}_t^{-j}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^{i_1} \middle| \mathbf{H}_t^{-j}\right) \Big] \times \\ & \times \mathcal{B}^1(H^{i_1}) + \Big[Prob\left(\mathbf{H}_{t_u} = H^{i_1} \middle| \mathbf{H}_t^{-j}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^{i_1} \middle| \mathbf{H}_t^{-j}\right) \Big] \mathcal{B}^2(H^{i_1}) \Big\} \end{aligned} \tag{135}$$

We now show that

$$\begin{aligned} & \lim_{n\to\infty} Prob\left(\mathbf{H}_{t_u} = H^{i_1} \middle| \mathbf{H}_t^{-i_1}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^{i_1} \middle| \mathbf{H}_t^{-i_1}\right) \\ & \qquad = - \lim_{n\to\infty} Prob\left(\mathbf{H}_{t_u} = H^{i_1} \middle| \mathbf{H}_t^{-i_2}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^{i_1} \middle| \mathbf{H}_t^{-i_2}\right) \end{aligned} \tag{136}$$

We proceed as with the proof of (127), starting from (81) – after the proper modifications: $k = 1$ and firm specific intensities – and cancelling terms 1 and 2, then cancelling terms 1-6 in (84), until we arrive at

$$p_t^{t_u}(H^{i_1}) - p_t^{t_u}(\overline{H^{i_1}}) \approx \text{RHS of (86)} \tag{137}$$

for n large. $\mathbf{H}_t^{-i_1}$ is of the form H^{i_1}, while $\mathbf{H}_t^{-i_2}$ is of the form $\overline{H}^{i_1}$, therefore letting $m = n^{\Delta}$ on the RHS of (86) it must be

$$\begin{aligned} & (*) \quad Prob\left(\mathbf{H}_{t_u} = H^{i_1} \middle| \mathbf{H}_t^{-i_1}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^{i_1} \middle| \mathbf{H}_t^{-i_1}\right) = \mathcal{T}^0(m, H^{i_1}) \\ & (**) \quad Prob\left(\mathbf{H}_{t_u} = H^{i_1} \middle| \mathbf{H}_t^{-i_2}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^{i_1} \middle| \mathbf{H}_t^{-i_2}\right) = -\mathcal{T}^0(m, \overline{H}^{i_1}) \\ & \text{or} \\ & (*) = \sum_{w_1=+1,-1} \sum_{\substack{v_1 \in \mathcal{S}^{w_1}(H^{i_1}) \\ v_1 \neq l(w_1)}} \mathcal{T}^1_{w_1}(m, H^{i_1}, v_1) \\ & \qquad\qquad (**) = - \sum_{w_1=+1,-1} \sum_{\substack{v_1 \in \mathcal{S}^{w_1}(\overline{H}^{i_1}) \\ v_1 \neq \overline{l}(w_1)}} \mathcal{T}^1_{w_1}(m, \overline{H}^{i_1}, v_1) \\ & \qquad\qquad \text{or} \\ & (*) = \sum_{w_1=+1,-1} \sum_{\substack{v_1 \in \mathcal{S}^{w_1}(H^{i_1}) \\ v_1 \neq l(w_1)}} \sum_{w_2=+1,-1} \sum_{\substack{v_2 \in \mathcal{S}^{w_2}(H^{i_1+w_1 v_1}) \\ v_2 \neq l(w_2)}} \mathcal{T}^2_{w_1,w_2}(m, H^{i_1}, v_1, v_2) \\ & (**) = - \sum_{w_1=+1,-1} \sum_{\substack{v_1 \in \mathcal{S}^{w_1}(\overline{H}^{i_1}) \\ v_1 \neq \overline{l}(w_1)}} \sum_{w_2=+1,-1} \sum_{\substack{v_2 \in \mathcal{S}^{w_2}(\overline{H}^{i_1+w_1 v_1}) \\ v_2 \neq \overline{l}(w_2)}} \mathcal{T}^2_{w_1,w_2}(m, \overline{H}^{i_1}, v_1, v_2) \\ & \text{or} \\ & \ldots \\ & \ldots \end{aligned} \tag{138}$$

Notice that any set of the form $\mathcal{S}^{w_1}(H^{i_1})$, excluding $l(w_1)$, coincides with $\mathcal{S}^{w_1}(\overline{H}^{i_1})$ excluding $\overline{l}(w_1)$. Notice also that $\lim_{n\to\infty} \mathcal{T}^0(m, H^{i_1}) = \lim_{n\to\infty} \mathcal{T}^0(m, \overline{H}^{i_1})$, $\lim_{n\to\infty} \mathcal{T}^1_{w_1}(m, H^{i_1}, v_1) = \lim_{n\to\infty} \mathcal{T}^1_{w_1}(m, \overline{H}^{i_1}, v_1)$, and so on: considering expressions (87)-(91), we notice that terms of the form $(1 - \exp(-\omega^i \Delta))$, $\omega = \lambda, \eta$ become independent of the specific firm i because of (122). We can conclude that (162) holds. Considering expression (135), it is clear that (162) and the fact that $\lambda^{i_1} \approx \lambda^{i_2}$ for n large (because of (62)) imply that

$$\lim_{n\to\infty} \mathcal{R}^{i_2} - \mathcal{R}^{i_1} = 0. \tag{139}$$

Thus

$$\lim_{n\to\infty} \left[\mu_t^{i_1} - \gamma\sigma^2 - \sum_{j=1}^{n} \tilde{\lambda}^j\right] \frac{P^{i_1}(\mathbf{H}_t)}{x_t^{i_1}(\mathbf{H}_t)} = \lim_{n\to\infty} \left[\mu_t^{i_2} - \gamma\sigma^2 - \sum_{j=1}^{n} \tilde{\lambda}^j\right] \frac{P^{i_2}(\mathbf{H}_t)}{x_t^{i_2}(\mathbf{H}_t)} \tag{140}$$

Since any two individual assets risk premia (not currently in distress) can be expressed asymptotically as a linear combination of each other, an exact one factor asymptotic structure holds for the expected returns of firms not currently in distress.

'Star' Network of Figure 2.
Firms' intensity parameters are independent only conditionally on a given state of aggregate (non) distress H. Thus parameters on the same row of the transition matrix $\mathbf{A}^H$ are mutually independent, but parameters on different rows are correlated. To model the 'star' network of Figure 2, let $\mathbf{H^1}$ denote the states where firm 1 (the central firm) is in distress, and $\overline{\mathbf{H^1}}$ the states where it is not. Then:

$$\begin{aligned}
\lambda^i(\mathbf{H^1}) &= k\lambda^i, \quad k>1, \quad i=2,\dots,n \qquad (141)\\
\lambda^i(\overline{\mathbf{H^1}}) &= \lambda^i \sim F^\lambda(n), \quad i=1,\dots,n \qquad (142)\\
\eta(\mathbf{H^1}) &= \eta(\overline{\mathbf{H^1}}) = \eta^i \sim F^\eta(n), \quad i=1,\dots,n \qquad (143)
\end{aligned}$$

Let N denote a generic noncentral firm. Then, from the previous case:

$$\begin{aligned}
\lim_{n\to\infty} \mathcal{R}^N_{ND} - \mathcal{R}^1_{ND} &= \lim_{n\to\infty} \sum_{\substack{j=1\\ j\neq 1,N}}^{n} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t) \sum_{u=0}^{\infty} e^{-a(t_u-t)} \sum_{H^1} \mathcal{U}(H^1,u)\\
\mathcal{U}(H^1,u) &= \left[kProb\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{\pm j}\right)\right]\epsilon^1(H^1)\\
&\quad + \left[Prob\left(\mathbf{H}_{t_u} = H^{i_1} \middle| \mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^{i_1} \middle| \mathbf{H}_t^{\pm j}\right)\right]\epsilon^2(H^1)\\
\epsilon^1(H^1) &= \begin{cases} 0 & \text{if} \quad \lim_{n\to\infty} \mathcal{ND}(H^1) = K < \infty \\ \epsilon^1 \sim N(\mu_1,\sigma_1) & \text{if} \quad \lim_{n\to\infty} \mathcal{ND}(H^1) = \infty \end{cases}\\
\epsilon^2(H^1) &= \begin{cases} 0 & \text{if} \quad \lim_{n\to\infty} \mathcal{D}(H^1) = K < \infty \\ \epsilon^2 \sim N(\mu_2,\sigma_2) & \text{if} \quad \lim_{n\to\infty} \mathcal{D}(H^1) = \infty \end{cases}
\end{aligned} \tag{144}$$

Because of Assumption 4, Lemma 4 of Proposition 3 holds in the present context. Let

$$\begin{aligned}
\epsilon^1_D = \lambda^N(\mathbf{H}_t)\theta^N(\mathbf{H}_t) \sum_{u=0}^{\infty} e^{-a(t_u-t)} &\left[\sum_H Prob\left(H_{t_u} = H \middle| \mathbf{H}_t^{-N}\right) \mathcal{A}^N(H)\right]\\
&- \lim_{n\to\infty} \lambda^1(\mathbf{H}_t)\theta^1(\mathbf{H}_t) \sum_{u=0}^{\infty} e^{-a(t_u-t)} \left[\sum_H Prob\left(H_{t_u} = H \middle| \mathbf{H}_t^{-1}\right) \mathcal{A}^1(H)\right]
\end{aligned} \tag{145}$$

Since N was an arbitrary noncentral firm, and all noncentral firms are identical, the random variable ϵ^1_D does not depend on N. The same reasoning applies to $\lim_{n\to\infty} \mathcal{R}^N_{ND} - \mathcal{R}^1_{ND}$. Then

$$\mathcal{R}^N - \mathcal{R}^1 = \mathcal{R}^N_{ND} - \mathcal{R}^1_{ND} + \epsilon^1_D = \varepsilon^1 \tag{146}$$

for n large, and

$$\lim_{n\to\infty}\left[\mu_t^1-\gamma\sigma^2-\sum_{j=1}^{n}\tilde{\lambda}^j\right]\frac{P^1(\mathbf{H}_t)}{x_t^1(\mathbf{H}_t)}=\lim_{n\to\infty}\left[\mu_t^N-\gamma\sigma^2-\sum_{j=1}^{n}\tilde{\lambda}^j\right]\frac{P^N(\mathbf{H}_t)}{x_t^N(\mathbf{H}_t)}+\varepsilon^1 \tag{147}$$

where ε depends only on firm 1 and it is imperfectly correlated with μ^1. Expression (147) shows that for the 'Star' network of Figure 2, an asymptotic three-fund separation holds.

Symmetrically Connected Network of Figure 1b.
In this case all firms are connected among each other, but the effect of a distress event on the rest of the firms does not depend on the specific firm that experiences the distress. All firms are 'central' in an homogeneous way. Without loss of generality, we model this network as follows:

$$\lambda^i(H) = \tilde{k}(H)\lambda^i, \quad i=1,2,\ldots,n \tag{148}$$

$$\lambda^i(H^{\text{nd}}) = \lambda^i \sim F^{\lambda}(n), \quad i=1,\ldots,n \tag{149}$$

$$\eta(H) = \eta(H^{\text{nd}})=\eta^i\sim F^{\eta}(n), \quad i=1,\ldots,n \tag{150}$$

$$\tilde{k}(H) = \begin{cases} k^{\text{num}(\mathcal{D}(H))} & \text{if} \quad \text{num}(\mathcal{D}(H))\le n^D \\ k^{n^D} & \text{otherwise}\end{cases} \qquad k>1, \tag{151}$$

If no firm is in distress (state H^{nd}), firm distress intensities are iid. If some firm is in distress, these intensities are compounded as many times as firms in distress at a common gross rate k, up to a maximum number n^D. As we are considering limiting behaviors as $n\to\infty$, and distress propagation needs to occur at bounded intensity, the boundedness assumption is necessary.

For two arbitrary firms 1 and N, since in the generic state H^1 and its paired $\overline{H}^1$ the same number of firms are in distress, expression (144) becomes:

$$\lim_{n\to\infty}\mathcal{R}_{ND}^N-\mathcal{R}_{ND}^1 = \lim_{n\to\infty}\sum_{\substack{j=1\\ j\neq 1,N}}^{n}(1+\tilde{k}(\mathbf{H}_t)\mathbf{1}(H_t^j=0))\tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-a(t_u-t)}\sum_{H^1}\mathcal{U}(H^1,u)$$

$$\begin{aligned}
\mathcal{U}(H^1,u) &= \tilde{k}(H^1)\left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\,\mathbf{H}_t^{\pm j}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\,\mathbf{H}_t^{\pm j}\right)\right]\epsilon^1(H^1)\\
&\quad+\left[Prob\left(\mathbf{H}_{t_u}=H^{i_1}\middle|\,\mathbf{H}_t^{\pm j}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^{i_1}\middle|\,\mathbf{H}_t^{\pm j}\right)\right]\epsilon^2(H^1)\\
\epsilon^1(H^1) &= \begin{cases}0 & \text{if} \quad \lim_{n\to\infty}\mathcal{ND}(H^1)=K<\infty\\ \epsilon^1\sim N(\mu_1,\sigma_1) & \text{if} \quad \lim_{n\to\infty}\mathcal{ND}(H^1)=\infty\end{cases}\\
\epsilon^2(H^1) &= \begin{cases}0 & \text{if} \quad \lim_{n\to\infty}\mathcal{D}(H^1)=K<\infty\\ \epsilon^2\sim N(\mu_2,\sigma_2) & \text{if} \quad \lim_{n\to\infty}\mathcal{D}(H^1)=\infty\end{cases},
\end{aligned} \tag{152}$$

We have

$$\lim_{n\to\infty}Prob\left(\mathbf{H}_{t_u}=H^1\middle|\,\mathbf{H}_t^{\pm j}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\,\mathbf{H}_t^{\pm j}\right)=0 \qquad j=1,\ldots,n \tag{153}$$

We refer to the proof of Lemma 4. Terms 1 and 2 of (81) become

$$\underbrace{p_t^{t_u-\Delta}(H^{1+1})\left(1-e^{-\tilde{k}(H^{1+1})\lambda^1\Delta}\right)-p_t^{t_u-\Delta}(H^{1+1})\left(1-e^{-\tilde{k}(H^{1+1})\lambda^N\Delta}\right)}_{1} \tag{154}$$
$$\underbrace{p_t^{t_u-\Delta}(H^{1-N})\left(1-e^{-\eta^N\Delta}\right)-p_t^{t_u-\Delta}(H^{1-N})\left(1-e^{-\eta^1\Delta}\right)}_{2}$$

For n large, (122) implies $\lambda^{i_2}\approx\lambda^{i_1}\approx\eta^{i_2}\approx\eta^{i_1}\approx 0$, which together with the boundedness assumption (151) implies term 1 $\approx$ term 2 ≈ 0. Terms 1 and 2 of (84) become

$$\underbrace{p_t^{t_u-2\Delta}(H^{1+1})\left[\left(1-e^{-\tilde{k}(H^1)\lambda^1\Delta}\right)e^{-\left[\sum_{v\in\mathcal{D}(H^1)}\eta^v+\sum_{v\in\mathcal{ND}(H^1)}\tilde{k}(H^1)\lambda^v\right]\Delta}\right.} \tag{155}$$
$$\underbrace{\left.-\left(1-e^{-\tilde{k}(H^1)\lambda^N\Delta}\right)e^{-\left[\sum_{v\in\mathcal{D}(H^1)}\eta^v+\sum_{v\in\mathcal{ND}(H^1)}\tilde{k}(H^1)\lambda^v\right]\Delta}\right]\lambda\Delta}_{1} \tag{156}$$
$$+\underbrace{p_t^{t_u-2\Delta}(H^{1-N})\left[\left(1-e^{-\eta^N\Delta}\right)e^{-\left[\sum_{v\in\mathcal{D}(H^1)}\eta^v+\sum_{v\in\mathcal{ND}(H^1)}\tilde{k}(H^1)\lambda^v\right]\Delta}\right.} \tag{157}$$
$$\underbrace{\left.-\left(1-e^{-\eta^1\Delta}\right)e^{-\left[\sum_{v\in\mathcal{D}(H^1)}\eta^v+\sum_{v\in\mathcal{ND}(H^1)}\tilde{k}(H^1)\lambda^v\right]\Delta}\right]\lambda\Delta}_{2} \tag{158}$$

In light of (122) and (151), terms 1 and 2 also vanish for n large. The same reasoning applies to terms 1-6 in expression (84), and to the term of this type that arise from further backward substitutions (see the proof of Lemma 4). The rest of the proof is unchanged. Since $p_t^t(H)=0$ for any H of the type H^1 and $\overline{H}^1$, by the assumption that firm 1 and N are not in distress in $\mathbf{H}_t^{\pm j}$, the limit (153) follows, and

$$\lim_{n\to\infty}\mathcal{R}_{ND}^N-\mathcal{R}_{ND}^1=0, \tag{159}$$

so that

$$\begin{aligned}\lim_{n\to\infty}\mathcal{R}^N-\mathcal{R}^1 \quad = \quad & \sum_{j=1,N}\tilde{k}(\mathbf{H}_t)\lambda^j\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-a(t_u-t)}\sum_{H^1}\left\{\tilde{k}(H^1)\left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\,\mathbf{H}_t^{-j}\right)\right.\right. && (160)\\ & -Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\,\mathbf{H}_t^{-j}\right)\Big]\mathcal{B}^1(H^1) \\ & \left.+\left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\,\mathbf{H}_t^{-j}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\,\mathbf{H}_t^{-j}\right)\right]\mathcal{B}^2(H^1)\right\} && (161)\end{aligned}$$

We now show that

$$\begin{aligned}&\lim_{n\to\infty}Prob\left(\mathbf{H}_{t_u}=H^{i_1}\middle|\,\mathbf{H}_t^{-1}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\,\mathbf{H}_t^{-1}\right)\\ &\qquad=-\lim_{n\to\infty}Prob\left(\mathbf{H}_{t_u}=H^1\middle|\,\mathbf{H}_t^{-N}\right)-Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\,\mathbf{H}_t^{-N}\right)\end{aligned} \tag{162}$$

We proceed as with the proof of (127), starting from (81) – after the proper modifications: $k=1$ outside of exponentials, firm specific intensities of distress $\lambda^i\tilde{k}(H_t)$ and recovery η^i – and cancelling terms 1 and 2, then cancelling terms 1-6 in(84), until we arrive at

$$p_t^{t_u}(H^{i_1})-p_t^{t_u}(\overline{H^{i_1}})\approx \text{RHS of (86)} \tag{163}$$

for n large. Term $\mathcal{T}^0(m,H)$ becomes

$$\mathcal{T}^0(m,H) = \begin{cases} 1 \text{ if } m=0 \\ \mathcal{T}^0(m-1,H)e^{-\left[\sum_{v\in\mathcal{D}(H)}\eta^v+\sum_{v\in\mathcal{ND}(H)}\tilde{k}(H)\lambda^v\right]\Delta} \quad \text{otherwise} \end{cases} \tag{164}$$

and similarly for higher order terms in (87)-(91). $\mathbf{H}_t^{-1}$ is of the form H^1, while $\mathbf{H}_t^{-N}$ is of the form $\overline{H}^1$, therefore letting $m=n^\Delta$ on the RHS of (86) it must be

$$\begin{aligned}
&(*) \quad Prob\left(\mathbf{H}_{t_u}=H^1\middle|\mathbf{H}_t^{-1}\right) - Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\mathbf{H}_t^{-1}\right) = \mathcal{T}^0(m,H^1)\\
&(**) \quad Prob\left(\mathbf{H}_{t_u}=H^1\middle|\mathbf{H}_t^{-N}\right) - Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\mathbf{H}_t^{-N}\right) = -\mathcal{T}^0(m,\overline{H}^1)\\
&\text{or}\\
&(*) = \sum_{w_1=+1,-1}\sum_{\substack{v_1\in\mathcal{S}^{w_1}(H^1)\\ v_1\neq l(w_1)}}\mathcal{T}^1_{w_1}(m,H^1,v_1)\\
&(**) = -\sum_{w_1=+1,-1}\sum_{\substack{v_1\in\mathcal{S}^{w_1}(\overline{H}^1)\\ v_1\neq \bar{l}(w_1)}}\mathcal{T}^1_{w_1}(m,\overline{H}^1,v_1)\\
&\text{or}\\
&(*) = \sum_{w_1=+1,-1}\sum_{\substack{v_1\in\mathcal{S}^{w_1}(H^1)\\ v_1\neq l(w_1)}}\sum_{w_2=+1,-1}\sum_{\substack{v_2\in\mathcal{S}^{w_2}(H^{1+w_1v_1})\\ v_2\neq l(w_2)}}\mathcal{T}^2_{w_1,w_2}(m,H^1,v_1,v_2)\\
&(**) = -\sum_{w_1=+1,-1}\sum_{\substack{v_1\in\mathcal{S}^{w_1}(\overline{H}^1)\\ v_1\neq \bar{l}(w_1)}}\sum_{w_2=+1,-1}\sum_{\substack{v_2\in\mathcal{S}^{w_2}(\overline{H}^{1+w_1v_1})\\ v_2\neq \bar{l}(w_2)}}\mathcal{T}^2_{w_1,w_2}(m,\overline{H}^1,v_1,v_2)\\
&\text{or}\\
&\ldots\\
&\ldots
\end{aligned} \tag{165}$$

Notice that any set of the form $\mathcal{S}^{w_1}(H^1)$, excluding $l(w_1)$, coincides with $\mathcal{S}^{w_1}(\overline{H}^1)$ excluding $\bar{l}(w_1)$. Notice also that $\lim_{n\to\infty}\mathcal{T}^0(m,H^1) = \lim_{n\to\infty}\mathcal{T}^0(m,\overline{H}^1)$, $\lim_{n\to\infty}\mathcal{T}^1_{w_1}(m,H^1,v_1) = \lim_{n\to\infty}\mathcal{T}^1_1(m,\overline{H}^1,v_1)$, and so on: considering expressions (87)-(91), we notice that terms of the form $(1-\exp(-\omega^i\Delta))$, $\omega=\tilde{k}(H)\lambda,\eta$ become independent of the specific firm i because of (122) and (151). We can conclude that (162) holds. Considering expression (135), it is clear that (162) and the fact that $\tilde{k}(\mathbf{H}_t)\lambda^1 \approx \tilde{k}(\mathbf{H}_t)\lambda^2$ for n large (because of (122)) imply that $\lim_{n\to\infty}\mathcal{R}^N \approx \lim_{n\to\infty}\mathcal{R}^1$, and that an exact conditional one factor asymptotic structure holds for expected returns of the assets that are not currently in distress.

Proof of Corollary 1. We use the notation of the proof of Proposition 3. We model a clustered economy with $\overline{n}$ central firms as follows: $\mathcal{G}$ is the collection of central firms; each firm $j\in\mathcal{G}$ is central for his own subnetwork, which is organized as in Figure 2. $\mathcal{NG}_j$ denotes the collection of noncentral firms in the subnetwork of j, with $(\mathcal{NG}_{j_1}\bigcap\mathcal{NG}_{j_2})=\varnothing$, $j_1,j_2\in\mathcal{G}$. Central firms are all symmetrically interconnected for simplicity, as in Figure 1b.

We summarize this description as follows:

$$\lambda^i(H) = \tilde{k}^i(H)\lambda^i, \quad i = 1, 2, \ldots, n \tag{166}$$

$$\lambda^i = \sim F^{\lambda}(n), \quad i = 1, \ldots, n \tag{167}$$

$$\eta(H) = \eta^i \sim F^{\eta}(n), \quad i = 1, \ldots, n \tag{168}$$

$$\tilde{k}^i(H) = \begin{cases} \prod_{v \in \mathcal{D}_c(H)} k_0 & \text{if} \quad i \in \mathcal{G} \\ k_j & \text{if} \quad i \in \mathcal{NG}_j \end{cases} \quad k_0, k_j > 1, \; j \in \mathcal{G}. \tag{169}$$

The number of central firms is independent of the economy size n. $\mathcal{D}_c(H)$ denotes the set of central firms that are in distress in state H. Therefore each central distress event compounds the distress risk of other central firms at some homogeneous rate k_0. Noncentral firms are affected only by the distress of their 'Star'. Assumption 3 and Assumption 4 hold.

To identify an asymptotic factor structure of expected returns, we relate the risk premia of any two firms of all possible types. As in Proposition 4 and Proposition 3, when we consider the risk premia of firms i and j, it is convenient to decompose the space of possible states H into: i) paired states $(H^i, \overline{H}^i)$ – i (j) is in distress in the former (latter), but not in the latter (former), while the state of all other firms coincide –; ii) states H where both i and j are not in distress; iii) states where they are both in distress. The following Lemma allows to concentrate only on cases i) and ii).

Lemma 6. *If firms i and j are both in distress in state H, then $\mathcal{A}^i(H) - \mathcal{A}^j(H) = 0$. If both are not in distress in H, then*

$$\mathcal{A}^i(H) - \mathcal{A}^j(H) = \left(\tilde{k}^i(H)\lambda^i - \tilde{k}^j(H)\lambda^j\right) \Delta \left(C^i(H^{-i}) - C^j(H^{-i})\right) \tag{170}$$

Proof. If i and j are both in distress, we have:

$$\begin{aligned} \mathcal{A}^i(H) - \mathcal{A}^j(H) = & \sum_{v \in \mathcal{ND}(H)} \tilde{k}^v(H)\lambda^v \Delta \left[c^i(H^{-v}) - c^j(H^{-v}) - \left(c^i(H) - c^j(H)\right)\right] \\ & - \sum_{\substack{v \in \mathcal{D}(H) \\ v \neq i,j}} \eta \Delta \left[c^i(H) - c^j(H) - \left(c^i(H^{+v}) - c^j(H^{+v})\right)\right] + (1-a)(c^i(H) - c^j(H)) \\ & - \underbrace{\eta \Delta \left[c^i(H) - c^j(H) - \left(c^i(H^{+i}) - c^j(H^{+i})\right)\right]}_{1} - \underbrace{\eta \Delta \left[c^i(H) - c^j(H) - \left(c^i(H^{+j}) - c^j(H^{+j})\right)\right]}_{2} \end{aligned} \tag{171}$$

$c^i(H) - c^j(H) = 0$, $c^i(H^{-v}) - c^j(H^{-v}) = 0$, $c^i(H^{+v}) - c^j(H^{+v}) = 0$, $v \neq i, j$, while terms 1 and 2 in (171) are opposite, so that $\mathcal{A}^i(H) - \mathcal{A}^j(H) = 0$. If i and j are both not in distress, we have:

$$\begin{aligned} \mathcal{A}^i(H) - \mathcal{A}^j(H) = & \sum_{\substack{v \in \mathcal{ND}(H) \\ v \neq i,j}} \tilde{k}^v(H)\lambda^v \Delta \left[c^i(H^{-v}) - c^j(H^{-v}) - \left(c^i(H) - c^j(H)\right)\right] \\ & - \sum_{v \in \mathcal{D}(H)} \eta \Delta \left[c^i(H) - c^j(H) - \left(c^i(H^{+v}) - c^j(H^{+v})\right)\right] + (1-a)(c^i(H) - c^j(H)) \\ & - \underbrace{\lambda^i \tilde{k}^i(H) \Delta \left[c^i(H) - c^j(H) - \left(c^i(H^{-i}) - c^j(H^{-i})\right)\right]}_{1} - \lambda^j \tilde{k}^j(H) \underbrace{\eta \Delta \left[c^i(H) - c^j(H) - \left(c^i(H^{-j}) - c^j(H^{-j})\right)\right]}_{2} \end{aligned} \tag{172}$$

Again $c^i(H) - c^j(H) = 0$, $c^i(H^{-v}) - c^j(H^{-v}) = 0$, $c^i(H^{+v}) - c^j(H^{+v}) = 0$, $v \neq i, j$. (170) follows adding terms 1 and 2 and taking into account that $c^i(H^{-i}) - c^j(H^{-i}) = -(c^i(H^{-j}) - c^j(H^{-j}))$. ☐

For ease of notation, we call firms 1 and N regardless of their type.

Firm 1 is central, Firm N is not

Let H_d denote a generic state where firms 1 and N are both in distress. Adapting (107) of Proposition 3 to the network characteristics reported in (166), keeping in mind the asymptotic homogeneity of dividends, and taking Lemma 6 into account:

$$\mathcal{R}^N_{ND} - \mathcal{R}^1_{ND} = \sum_{\substack{v=j \\ j\neq 1,N}}^{n} \tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t) \sum_{u=0}^{\infty} e^{-a(t_u - t)} \left(\sum_{H^1} \mathcal{U}^1(H^1, u) + \sum_{H_d} \mathcal{U}^2(H_d, u) \right) \tag{173}$$

$$\begin{aligned}
\mathcal{U}(H^1,u) = \sum_{\substack{u\in\mathcal{G}\\u\neq 1}} \sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\in\mathcal{NG}_u \\ v\neq N}} & \underbrace{\tilde{k}^v(H^1)\lambda^v \left[Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{\pm j}\right)\right] \Delta \left(c^N(H^{1-v}) - c^1(H^{1-v}) - \right.}_{} \\
& \underbrace{\left(c^N(H^1) - c^1(H^1)\right)\Big)}_{1} + \sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\in\mathcal{NG}_1 \\ v\neq N}} \underbrace{\left[k_1 Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{\pm j}\right)\right] \lambda^v \Delta \left(c^N(H^{1-v}) - c^1(H^{1-v})\right.}_{} \\
& \underbrace{-\left(c^N(H^1) - c^1(H^1)\right)\Big)}_{2} - \underbrace{\left[Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{\pm j}\right)\right]}_{} \times \\
& \times \underbrace{\left(\lambda^N \tilde{k}^N(H^1) - \lambda^1 \tilde{k}^1(H^1)\right) \left(c^N(H^1) - c^1(H^1)\right)\Delta}_{3} \\
& + \sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\in\mathcal{G} \\ v\neq 1}} \underbrace{\tilde{k}^v(H^1)\lambda^v \left[Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{\pm j}\right)\right] \Delta \left(c^N(H^{1-v}) - c^1(H^{1-v}) - \right.}_{} \\
& \underbrace{\left(c^N(H^1) - c^1(H^1)\right)\Big)}_{4} + \underbrace{\left[Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u} = \overline{H}^1 \middle| \mathbf{H}_t^{\pm j}\right)\right]}_{} \times \\
& \times \underbrace{\Big[(1-a)(c^N(H^1) - c^1(H^1)) - \Delta\left(\eta^N c^N(H^1) - \eta^1 c^1(H^1)\right)}_{} \\
& \underbrace{+ \sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}} \eta^v \Delta \left(\left(c^N(H^{1+v}) - c^1(H^{1+v})\right) - \left(c^N(H^1) - c^1(H^1)\right)\right)\Bigg]}_{5}
\end{aligned}$$

$$\mathcal{U}^2(H_d,u) = Prob\left(\mathbf{H}_{t_u} = H_d \middle| \mathbf{H}_t^{\pm j}\right) \left(\tilde{k}^N(H_d)\lambda^N - \tilde{k}^1(H_d)\lambda^1\right) \Delta \left(c^N(H_d^{-N}) - c^1(H_d^{-N})\right)$$

We note that $\tilde{k}^i(H) < \infty$, $\forall n$, $i = 1, \ldots, n$, because the number of central firms, $\overline{n}$, is bounded by assumption. Notice that the characteristics of the specific subnetwork of firm N enter only in term 3, which is o(term 2) as $n \to \infty$. By Assumptions 3 and 4, and the fact that $Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{\pm j}\right)$ and $Prob\left(\mathbf{H}_{t_u} = H^1 \middle| \mathbf{H}_t^{\pm j}\right)$ converge to a deterministic limit as $n \to \infty$, terms 1, 2, 4, 5 are sums of independent random variables. We also assume that the Linderg-Feller conditon holds. The Central Limit Theorem then guarantees asymptotic convergence of the sum of these terms to a random variables which only depends on firm 1 characteristics, because of the asymptotic behavior of term 3. Thus

$$\lim_{n\to\infty}\sum_{H^1}\mathcal{U}^1(H^1,u)=\epsilon^1_{u,j}(1) \tag{174}$$

Due to Assumptions 1 and 4, $\sum_{H_d}\mathcal{U}^2(H_d,u)$ is a sum of independent random variables. By the Central Limit Theorem argument already applied in Proposition 4:

$$\lim_{n\to\infty}\sum_{H_d}\mathcal{U}^2(H_d,u)=\epsilon^2_{u,j}(1,N)\sim N(\mu^2_{u,j},\sigma^2_{u,j}) \tag{175}$$

The random variable $\epsilon^2_{u,j}(1,N)$ in general takes positive and negative values with nonzero probability. Its distribution depends on the centrality parameters ks. Thus, by means of (173), and again the Central Limit Theorem argument

$$\mathcal{R}^N_{ND}-\mathcal{R}^1_{ND}\approx\sum_{\substack{v=j\\ j\neq 1,N}}^{n}\tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-a(t_u-t)}\left(\epsilon^1_{u,j}(1)+\epsilon^2_{u,j}(1,N)\right) \tag{176}$$

for n large. (176) then implies that

$$\lim_{n\to\infty}\mathcal{R}^N-\mathcal{R}^1=\lim_{n\to\infty}\mathcal{R}^N_{ND}-\mathcal{R}^1_{ND}+\sum_{j=1,N}\tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-a(t_u-t)}\left(\sum_{H^1}\mathcal{U}^1(H^1,u)+\sum_{H_d}\mathcal{U}^2(H_d,u)\right) \tag{177}$$

Due to Assumption 4 $\tilde{\lambda}^1\approx\tilde{\lambda}^N\approx 0$ for n large, while $\left(\sum_{H^1}\mathcal{U}^1(H^1,u)+\sum_{H_d}\mathcal{U}^2(H_d,u)\right)$ converge to a random variable that is bounded $P-$a.s, by virtue of Assumptions 3 and 4. We conclude that

$$\lim_{n\to\infty}\mathcal{R}^N-\mathcal{R}^1=\sum_{\substack{v=j\\ j\neq 1,N}}^{n}\tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-a(t_u-t)}\left(\epsilon^1_{u,j}(1)+\epsilon^2_{u,j}(1,N)\right) \tag{178}$$

Firm 1 is central, Firm N is central (173) becomes

$$\mathcal{R}^N_{ND}-\mathcal{R}^1_{ND} = \sum_{\substack{v=j\\ j\neq 1,N}}^{n}\tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-a(t_u-t)}\left(\sum_{H^1}\mathcal{U}^1(H^1,u)+\sum_{H_d}\mathcal{U}^2(H_d,u)\right) \tag{179}$$

$$\mathcal{U}(H^1,u) = \underbrace{\sum_{\substack{u\in\mathcal{G}\\ u\neq 1,N}} \sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\in\mathcal{NG}_u}} \tilde{k}^v(H^1)\lambda^v \left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\mathbf{H}_t^{\pm j}\right)\right]\Delta\left(c^N(H^{1-v}) - c^1(H^{1-v})-\right.}$$

$$\underbrace{\left.\left(c^N(H^1)-c^1(H^1)\right)\right)}_{1} + \underbrace{\sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\in\mathcal{NG}_1}} \left[k_1 Prob\left(\mathbf{H}_{t_u}=H^1\middle|\mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\mathbf{H}_t^{\pm j}\right)\right]\lambda^v\Delta\left(c^N(H^{1-v}) - c^1(H^{1-v})\right.}$$

$$\underbrace{\left.-\left(c^N(H^1)-c^1(H^1)\right)\right)}_{2} + \underbrace{\sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\in\mathcal{NG}_N}} \left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\mathbf{H}_t^{\pm j}\right) - k_N Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\mathbf{H}_t^{\pm j}\right)\right]\lambda^v\Delta\left(c^N(H^{1-v}) - c^1(H^{1-v})\right.}$$

$$\underbrace{\left.-\left(c^N(H^1)-c^1(H^1)\right)\right)}_{3} - \underbrace{\left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\mathbf{H}_t^{\pm j}\right)\right]\left(\lambda^N\tilde{k}^N(H^1) - \lambda^1\tilde{k}^1(H^1)\right)\times}$$

$$\underbrace{\times\left(c^N(H^1)-c^1(H^1)\right)\Delta}_{4} + \underbrace{\sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\in\mathcal{G}\\ v\neq 1,N}} \tilde{k}^v(H^1)\lambda^v\left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\mathbf{H}_t^{\pm j}\right)\right]\Delta\times}$$

$$\underbrace{\times\left(c^N(H^{1-v}) - c^1(H^{1-v}) - \left(c^N(H^1)-c^1(H^1)\right)\right)}_{5} + \underbrace{\left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\mathbf{H}_t^{\pm j}\right)\right]\times}$$

$$\times\underbrace{\left[(1-a)(c^N(H^1)-c^1(H^1)) - \Delta\left(\eta^N c^N(H^1) - \eta^1 c^1(H^1)\right)\right.}$$

$$\underbrace{\left.+\sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}} \eta^v\Delta\left(\left(c^N(H^{1+v}) - c^1(H^{1+v})\right) - \left(c^N(H^1)-c^1(H^1)\right)\right)\right]}_{6}$$

$$\mathcal{U}^2(H_d,u) = Prob\left(\mathbf{H}_{t_u}=H_d\middle|\mathbf{H}_t^{\pm j}\right)\left(\tilde{k}^N(H_d)\lambda^N - \tilde{k}^1(H_d)\lambda^1\right)\Delta\left(c^N(H_d^{-N}) - c^1(H_d^{-N})\right)$$

Due to terms 2 and 3, Assumptions 3 and 4 allow to apply the Central Limit Theorem, which guarantees convergence of $\mathcal{U}^1(H^1,u)$ to a random variable – or more correctly, to a sum of random variables – that depends on the characteristics of the central firms 1 and N. Taking into account that $\lambda^1 \approx \lambda^N \approx 0$ for n large, we have :

$$\begin{aligned}
\lim_{n\to\infty}\mathcal{R}^N - \mathcal{R}^1 &= \lim_{n\to\infty}\mathcal{R}^N_{ND} - \mathcal{R}^1_{ND} + \sum_{j=1,N}\tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-a(t_u-t)}\sum_{H^1}\mathcal{U}^1(H^1,u) \\
&= \lim_{n\to\infty}\mathcal{R}^N_{ND} - \mathcal{R}^1_{ND} = \sum_{\substack{v=j\\ j\neq 1,N}}^{n}\tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-a(t_u-t)}\epsilon^3_{u,j}(1,N)
\end{aligned} \tag{180}$$

Firm 1 is not central, Firm N is not central (173) becomes

$$\mathcal{R}^N_{ND} - \mathcal{R}^1_{ND} = \sum_{\substack{v=j\\ j\neq 1,N}}^{n}\tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-a(t_u-t)}\left(\sum_{H^1}\mathcal{U}^1(H^1,u) + \sum_{H_d}\mathcal{U}^2(H_d,u)\right) \tag{181}$$

$$\mathcal{U}(H^1,u) = \underbrace{\sum_{\substack{v\in\mathcal{ND}(H^1)\\ v\neq N}} \tilde{k}^v(H^1)\lambda^v \left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\,\mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\,\mathbf{H}_t^{\pm j}\right)\right]\Delta\left(c^N(H^{1-v})-c^1(H^{1-v})-\right.}$$

$$\underbrace{\left.\left(c^N(H^1)-c^1(H^1)\right)\right)}_{1} - \underbrace{\left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\,\mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\,\mathbf{H}_t^{\pm j}\right)\right]\left(\lambda^N\tilde{k}^N(H^1)-\lambda^1\tilde{k}^1(H^1)\right)\left(c^N(H^1)-c^1(H^1)\right)\Delta}_{2}$$

$$+\underbrace{\left[Prob\left(\mathbf{H}_{t_u}=H^1\middle|\,\mathbf{H}_t^{\pm j}\right) - Prob\left(\mathbf{H}_{t_u}=\overline{H}^1\middle|\,\mathbf{H}_t^{\pm j}\right)\right]\left[(1-a)(c^N(H^1)-c^1(H^1)) - \Delta\left(\eta^N c^N(H^1)-\eta^1 c^1(H^1)\right)\right.}$$

$$\underbrace{\left.+\sum_{\substack{v\in\mathcal{D}(H^1)\\ v\neq 1}} \eta^v\Delta\left(\left(c^N(H^{1+v})-c^1(H^{1+v})\right)-\left(c^N(H^1)-c^1(H^1)\right)\right)\right]}_{3}$$

$$\mathcal{U}^2(H_d,u) = Prob\left(\mathbf{H}_{t_u}=H_d\middle|\,\mathbf{H}_t^{\pm j}\right)\left(\tilde{k}^N(H_d)\lambda^N-\tilde{k}^1(H_d)\lambda^1\right)\Delta\left(c^N(H_d^{-N})-c^1(H_d^{-N})\right)$$

Notice that term 2 is o(term 1) for $n\to\infty$, while applying previous arguments, terms 1 and 3 converge to a random variable that doesn't depend on the firm 1 and N's subnetwork. On the other hand:

$$\lim_{n\to\infty}\sum_{H_d}\mathcal{U}^2(H_d,u) = \epsilon^4_{u,j}(1,N)\sim N(\mu^4_{u,j},\sigma^4_{u,j}) \tag{182}$$

where $\epsilon^4_{u,j}(1,N)$ in nonnegative $P-a.s.$ if $k_N\leq k_1$. Summarizing:

$$\begin{aligned}
\lim_{n\to\infty}\mathcal{R}^N-\mathcal{R}^1 &= \lim_{n\to\infty}\mathcal{R}^N_{ND}-\mathcal{R}^1_{ND} + \sum_{j=1,N}\tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-a(t_u-t)}\left(\sum_{H^1}\mathcal{U}^1(H^1,u)+\sum_{H_d}\mathcal{U}^2(H_d,u)\right)\\
&= \lim_{n\to\infty}\mathcal{R}^N_{ND}-\mathcal{R}^1_{ND} = \sum_{\substack{v=j\\ j\neq 1,N}}^{n}\tilde{\lambda}^j(\mathbf{H}_t)\theta^j(\mathbf{H}_t)\sum_{u=0}^{\infty}e^{-a(t_u-t)},\epsilon^4_{u,j}(1,N)
\end{aligned} \tag{183}$$

after taking into account that, due to Assumption 4, $\tilde{\lambda}^1\approx\tilde{\lambda}^N\approx 0$ for n large, while $\left(\sum_{H^1}\mathcal{U}^1(H^1,u)+\sum_{H_d}\mathcal{U}^2(H_d,u)\right)$ converge to a random variable that is bounded $P-$a.s.

Substituting (178), then (180), then (183) into expression (125) for the risk premium, we realize that the risk premium of firm i, can be expressed as a linear combination of the risk premium of firm j and of an additional random variable $\varepsilon(i,j)$

$$\lim_{n\to\infty}\left[\mu^i_t-\gamma\sigma^2-\sum_{v=1}^{n}\tilde{\lambda}^v\right]\frac{P^i(\mathbf{H}_t)}{x^i_t(\mathbf{H}_t)} = \lim_{n\to\infty}\left[\mu^j_t-\gamma\sigma^2-\sum_{v=1}^{n}\tilde{\lambda}^v\right]\frac{P^j(\mathbf{H}_t)}{x^j_t(\mathbf{H}_t)}+\varepsilon(i,j) \tag{184}$$

$\varepsilon(i,j)$ depends on firm i's and j's type: on the specific firms if they are central, on their subnetwork if they are not central. Since any two $\varepsilon(i_1,j_1)$ and $\varepsilon(i_2,j_2)$ are imperfectly correlated, there are $\overline{n}$ central firms and $\overline{n}$ subnetworks, any risk premium can be expressed as a linear combination of $2\overline{n}$ other risk premia, and a $2\overline{n}+1$-factor representation holds.

Appendix B

Calibration procedure in Section V. We use historical data on real output of US domestic industries, from 1972 to 2010, published by the US Bureau of Labor Statistics. We group data into the $n = 9$ sectors reported in Table IX. We combine this information with the industry-by-industry total requirements table for 2010, part of the NIPA Tables published by the Bureau of Economic Analysis. The industry-by-industry requirement is the production required, both directly and indirectly, from some industry industry, per dollar of delivery to final use of a given industry. We assume that the percentage requirements in 2010 are representative of the whole sample. Parameters are calibrated as follows:

- The transition intensities of the business cycle factor are $k_l = 1.2616$ and $k_h = 0.3911$, obtained as described in Section VI.B.
- We build the network connections as follows. Let $\lambda_0^i = \lambda^i(0, \mathbf{H}^0)$, $i = 1, 2, \ldots, n+1$ denote the distress intensity of firm i in the boom state, when no firm is in distress, as $\mathbf{H}^0 = (0, 0, \ldots, 0)'$. If a sector experiences a distress, the other sectors' intensities increase, relative to the disconnected case, in percentage of the fraction of their total output required from the distressed sector. The effect is additive, meaning that when more sectors are in distress we sum the individual requirements to obtain the corresponding increase in λ. In other words:

$$\lambda^i(0, \mathbf{H}) = \lambda_0^i \left(1 + \sum_{j=1, j\neq i}^{n} H^i \frac{re(j,i)}{\sum_{u=1^n} re(u,i)}\right) \quad i = 1, n \tag{185}$$

  where $re(j,i)$ denotes the requirement of industry i from industry j. We assume for simplicity that the last sector – comprising the remaining ones – is not connected, so that its distress intensity is $\lambda^{10}(0, \mathbf{H}) = \lambda_0^{10}$, $\forall\, \mathbf{H}$.

  During a recession, the distress intensity increases by 25%: $\lambda^i(1, \mathbf{H}) = 1.25\lambda^i(0, \mathbf{H}^0)$. For simplicity, the recovery intensities are $\eta^i(0, \mathbf{H}) = \lambda^i(1, \mathbf{H}^0)$, and $\eta^i(1, \mathbf{H}) = \lambda^i(0, \mathbf{H}^0)$.
- The intensities conditional on no distress, λ_0^i, the logarithmic dividend jump sizes, J^i, $i = 1, 2, \ldots, n+1$, and the mean (μ) and volatility (σ) of the common diffusive component Y, remain to be determined. To this end, we use the method moments with $2n + 4$ moment conditions, as many as unknown parameters. The moments used are the unconditional yearly dividend growth (in logarithms), the unconditional variance of yearly dividend growth, for each of the $n+1$ endowments, and the unconditional third central moment of dividend growth only for the first two industries. The theoretical moments read:

$$\begin{aligned} m_1^i = \mathbf{E}\left[\log D_{t+1y}^i - \log D_t^i\right] = \pi'\mathbf{E}\left[\log D_{t+1y}^i - \log D_t^i \middle| \mathbf{H}\right] = \left(\mu - \frac{1}{2}\sigma^2\right) \\ + \pi'\mathbf{E}\left[\int_t^{t+1y} \left(J^i H_u^i dH_u^i - J^i(1 - H_u^i)\right) dH_u^i \middle| \mathbf{H}_t\right] = \left(\mu - \frac{1}{2}\sigma^2\right) + \\ \pi'\mathbf{E}\left[\int_t^{t+1y} \left(J^i H_u^i \eta^i(S_u, \mathbf{H}_u) - J^i(1 - H_u^i)\lambda^i(S_u, \mathbf{H}_u)\right) du \middle| \mathbf{H}_t\right] = \left(\mu - \frac{1}{2}\sigma^2\right) + \pi' \int_t^{t+1y} \exp\left(-\mathbf{A}^H(s-t)\right) ds \mathbf{C}^\lambda \\ i = 1, 2, \ldots, n+1 \end{aligned} \tag{186}$$

$$m_z^i = \mathbf{E}\left[(\log D_{t+1y}^i - \log D_t^i - m_1^i)^z\right] = \pi'\mathbf{E}\left[(\log D_{t+1y}^i - \log D_t^i - m_1^i)^z \,\middle|\, \mathbf{H}\right] =$$
$$\sigma^2 \mathbf{1}(z=2) + \pi'\mathbf{E}\left[\int_t^{t+1y} \left((J^i)^z H_u^i dH_u^i + (1/J^i)^z(1-H_u^i)\right) dH_u^i \,\middle|\, \mathbf{H}_t\right] =$$
$$\sigma^2 \mathbf{1}(z=2) + \pi'\mathbf{E}\left[\int_t^{t+1y} \left((J^i)^z H_u^i \eta^i(S_u, \mathbf{H}_u) - (1/J^i)^z(1-H_u^i)\lambda^i(S_u, \mathbf{H}_u)\right) du \,\middle|\, \mathbf{H}_t\right] =$$
$$\sigma^2 \mathbf{1}(z=2) + \pi' \int_t^{t+1y} \exp\left(-\mathbf{A}^H (s-t)\right) ds \mathbf{C}_z^\lambda \quad z = 2,3 \tag{187}$$

where we have used the dividend dynamics (3) to explicit logarithmic dividend growth. The transition matrix $-\mathbf{A}^H$ is reported in (41). π is the vector of steady state probabilities for the firms implies by $-\mathbf{A}^H$, as detailed in (55). $\mathbf{C}^\lambda$ is a $2 \times 2^{n+1}$ vector, with entry corresponding to state H and $S = 0$ given by $\left(J^i H_u^i \eta^i(0, H) - J^i(1 - H_u^i)\lambda^i(0, H)\right)$. The entry corresponding to H and $S = 1$ immediately follows the latter. $\mathbf{C}_z^\lambda$ is similarly defined, with entries: $\left((J^i)^z H_u^i \eta^i(S, H) + (1/J^i)^z(1 - H_u^i)\lambda^i(S, H)\right)$. Thus the set of moment conditions read

$$\left(m_1^i - \frac{1}{m}\sum_{j=2}^{m} \log \hat{D}_j^i - \log \hat{D}_{j-1}^i \,,\, m_z^i - \frac{1}{m}\sum_{j=2}^{m} \left(\log \hat{D}_j^i - \log \hat{D}_{j-1}^i - \hat{m}_1^i\right)^z \right)$$
$$(i = 1, 2, \ldots, n+1 \text{ for } z = 2), (i = 1, 2 \text{ for } z = 3) \tag{188}$$

where $\hat{D}$ denotes a sector output series, and $\hat{m}_1^i$ is the sample mean of logarithmic output growth.

- When we simulate asset prices from this economy, we set the risk aversion to $\gamma = 3.5$, and the subjective discount rate to $\delta = 0.03$.

Estimation procedure. Given trees (i, j), under the agent's observation filtration their dividend processes follow

$$\frac{dD_t^z}{D_t^z} = \mu^z dt + \sigma^z dB_t^z + (e^{-Y_z^1} - 1)(1 - H_t^z) dH_t^z - (e^{Y_z^2} - 1) H_t^z dH_t^z \quad z = i, j \quad dB_t^i dB_t^j = \rho \tag{189}$$

H_t^z is a two-state continuous-time Markov chain with transition intensities

$$\hat{\lambda}_t^z = p_t^h \lambda^z(0, H_t^i, H_t^j) + (1 - p_t^h)\lambda^z(1, H_t^i, H_t^j) \tag{190}$$
$$\hat{\eta}_t^z = p_t^h \eta^z(0, H_t^i, H_t^j) + (1 - p_t^h)\eta^z(1, H_t^i, H_t^j) \tag{191}$$

under the posterior belief. The latter evolves as

$$dp_t^h = \left[k_l - (k_l + k_h)p_t^h\right] dt + p_t^h(1 - p_t^h) \sum_{z=i,j} \left[\lambda^z(1, H_{t-}^i, H_{t-}^j) - \lambda^z(0, H_{t-}^i, H_{t-}^j)\right] \times$$
$$\times (1 - H_{t-}^z)(dH_t^z - \hat{\lambda}^z) + \left[\eta^z(1, H_{t-}^i, H_{t-}^j) - \eta^z(0, H_{t-}^i, H_{t-}^j)\right] H_{t-}^z (dH_t^z - \hat{\eta}^z) \tag{192}$$

Let $nd(\omega, t, t+1)$ (nr) denote the number of distress (recovery) jumps occurred in sample path ω between t and $t + 1$. Conditional on this sample path, the random variables

$$y_{t+1}^z = \log D_{t+1}^z - \log D_t^z - \left(\mu^z - \frac{\sigma^2}{2}\right)\frac{1}{4} + \left(\sum^{nd(\omega,t,t+1)} -Y_z^1 + \sum^{nr(\omega,t,t+1)} Y_z^2\right) \quad z = i, j \tag{193}$$

are jointly distributed as:

$$(y_{t+1}^i, y_{t+1}^j | \omega) \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^{i2} & \sigma^i\sigma^j\rho \\ \sigma^i\sigma^j\rho & \sigma^{j2} \end{bmatrix} \frac{1}{4}\right) \tag{194}$$

We don't observe the distress events nor the posterior beliefs that determine their intensities in the agent's filtration. Knowing agent's updating rule (192), and for a prior belief $p_0 = k_h/(k_h + k_l)$ equal to the stationary probability of a recession, we fix a given parameter set and simulate a trajectory of distress/recovery events and beliefs. We estimate parameters by maximum likelihood using the likelihood of the filtered continuous component of dividend growth, obtained subtracting the simulated events out of dividend growth. Thus we infer the posterior belief of the agent as the simulated trajectory ω corresponding to the following maximum likelihood parameter estimate.[24]

$$\theta^* = \arg\max_{\theta} \quad \sum_{t=0}^{N-1} \log \phi(\hat{y}_t^{i+1}, \hat{y}_{t+1}^j | \omega, \theta)$$

where $\phi(\hat{y}_t^i, \hat{y}_t^j | \omega, \theta)$ is the bivariate pdf (194) evaluated at the data, for a given parameter set θ, and a trajectory of distresses and beliefs ω simulated under the parameter set θ with standard schemes.[25]

Standard Errors. We obtain parameters' standard errors using the asymptotic distribution of the estimates, following the arguments of Brandt and Santa-Clara (2002):

- Let $\phi^{\Delta}(y_{t+1}^i, y_{t+1}^j | \omega, \theta)$ denote the transition density of (y_{t+1}^i, y_{t+1}^j) in (193), when beliefs and log dividend growth jumps follow the discretized process (197) and (198)-(199) used to simulate them, with time step size Δ:

$$y_{t+1}^z = \log D_{t+1}^z - \log D_t^z - (\mu^z - \frac{\sigma_z^2}{2})\frac{1}{4} - \left(\sum^{nd_z(\omega,t,t+1)} -Y_1^z + \sum^{nr_z(\omega,t,t+1)} Y_2^z \right)$$

$$z = i, j \quad t = 0, 1, \dots N-1 \quad (196)$$

$$p_{s+1}^h = \left[k_l - (k_l + k_h)p_s^h\right]\Delta + p_s^h(1 - p_s^h)\sum_{z=i,j} (\lambda^z(1, H_s^i, H_s^j) - \lambda^z(0, H_s^i, H_s^j))(1 - H_s^z) \times$$
$$\times \left[\mathbf{1}\left(\exp(-\hat{\lambda}^z\Delta) < u_s\right) - \hat{\lambda}^z\right] + (\eta^z(1, H_s^i, H_s^j) - \eta^z(0, H_s^i, H_s^j))H_s^z \times$$
$$\times \left[\mathbf{1}\left(\exp(-\hat{\eta}^z\Delta) < u_s\right) - \hat{\eta}^z\right] \quad (197)$$

  N is the number of quarters in the sample. Since a quarter separates t and $t+1$, and a year is discretized into $1/\Delta$ steps, s ranges from 0 to $N/(4\Delta)$. u_s is iid uniformly distributed on [0,1]. The posterior intensities of distress and recovery at time s read $\hat{\lambda}_s^z = p_s\lambda^z(0, H_s^i, H_s^j) + (1-p_s)\lambda^z(1, H_s^i, H_s^j)$ and $\hat{\eta}_s^z = p_s\eta^z(0, H_s^i, H_s^j) + (1-p_s)\eta^z(1, H_s^i, H_s^j)$. $nd_z(\omega, t, t+1)$ and $nr_z(\omega, t, t+1)$, the number of distress and recoveries of tree $z = i, j$

[24]We have also inspected a methodology that selects the belief and distress trajectory as the most likely to have generated the dividend observations, among as many as possible. Namely the maximum likelihood parameter estimate is:

$$\theta^* = \arg\max_{\theta} \quad \max_{w^i} \sum_{t=1}^{N} \log \phi(\hat{y}_t^i, \hat{y}_t^j | \omega^i, \theta) \quad (195)$$

This approach basically treats the belief and distress sample realization as an infinite dimensional parameter, constrained to be generated by a given distribution. It is computationally cumbersome and, according to limited investigation, has led to similar results.

[25]Note that except for k_h and k_l, which are separaterly determined , no parameter is common to any pairwise estimation, since the intensities of distress and recovery are contingent to the state of the other tree in the procedure, and all tree couples are distinct.

occurred in sample path ω between t and $t+1$, read:

$$nd_z(\omega, t, t+1) \quad = \quad \sum_{s=t}^{t+\frac{1}{4\Delta}} (1 - H_s^z)\mathbf{1}\left(\exp(-\hat{\lambda}^z\Delta) < u_s\right) \tag{198}$$

$$nr_z(\omega, t, t+1) \quad = \quad \sum_{s=t}^{t+\frac{1}{4\Delta}} H_s^z \mathbf{1}\left(\exp(-\hat{\eta}^z\Delta) < u_s\right) \tag{199}$$

The convergence of discretization schemes for stochastic differential equations (see Glasserman (2004)) implies that

$$plim_{\Delta t \to 0}\, \phi^{\Delta}(y_{t+1}^i, y_{t+1}^j|\omega, \theta) \quad = \quad \phi(y_{t+1}^i, y_{t+1}^j|\omega, \theta) \tag{200}$$

$$\equiv \quad N\left(\begin{bmatrix} (\mu_i - \frac{\sigma_i^2}{2})\frac{1}{4} \\ (\mu_j - \frac{\sigma_j^2}{2})\frac{1}{4} \end{bmatrix}, \begin{bmatrix} \sigma_i^2 & \sigma_i\sigma_j\rho \\ \sigma_i\sigma_j\rho & \sigma_j^2 \end{bmatrix}\frac{1}{4}\right) \tag{201}$$

where the last term is the transition density of (y_{t+1}^i, y_{t+1}^j) in (193) when beliefs and log dividend growth jumps follow their true continuous-time process.

- Let ω denote any sample path and ω^* denote the true, unobservable history of distress/recovery events and beliefs. The strong stationarity of the log dividend growth process $d\log(D_t)$, with dD_t as in (189), and of the posterior belief process (192), implies that

$$plim_{t \to \infty}\, \phi(y_{t+1}^i, y_{t+1}^j|\omega, \theta) = \phi(y_{t+1}^i, y_{t+1}^j|\omega^*, \theta) \tag{202}$$

  where the rhs denotes the likelihood corresponding to the true to the true (observable) distress/belief realizations. Moreover, since (y_{t+1}^i, y_{t+1}^j) is iid in time, by the Central Limit Theorem as $t \to \infty$ we have

$$\sqrt{t}[\phi(y_{t+1}^i, y_{t+1}^j|\omega, \theta) - \phi(y_{t+1}^i, y_{t+1}^j|\omega^*, \theta)] \sim N(0, var[\phi(y_t^i, y_t^j|\omega^i, \theta)]))$$

- By virtue of (201) and (202), we have

$$plim_{t \to \infty, \Delta t \to 0}\, \phi^{\delta}(y_{t+1}^i, y_{t+1}^j|\omega, \theta) = \phi(y_{t+1}^i, y_{t+1}^j|\omega^*, \theta) \tag{203}$$

- According to Lemma 4 and Lemma 5 in Brandt and Santa-Clara (2002), asymptotic convergence of the likelihood function can be shown, namely:

$$plim_{N \to \infty, \Delta t \to 0} \sum_{t=0}^{N} \log \phi^{\Delta}(y_t^i, y_t^j|\omega, \theta) = \sum_{t=0}^{N} \log \phi(y_t^i, y_t^j|\omega^*, \theta)$$

  Furthermore, the parameter set maximizing the simulated likelihood, $\hat{\theta}^{\Delta}$, converges in probability to the maximizer of the true likelihood, $\hat{\theta}$.

- We can then rely on the asymptotic theory of the full information case to show that, as $N \to \infty$ and $\Delta t \to 0$

$$(\hat{\theta} - \theta_0) \sim N(0, I^{-1}(\theta_0)), \quad I(\theta_0) = \mathbb{E}\left[\sum_{t=0}^{N} \frac{\partial \log \phi(y_{t+1}^i, y_{t+1}^j | \omega^*, \theta)}{\partial \theta} \frac{\partial \log \phi(y_{t+1}^i, y_{t+1}^j | \omega^*, \theta)}{\partial \theta}'\right]$$

  where I is the Fisher information matrix.

To estimate $I(\hat{\theta})$, we obtain the gradient sample path corresponding to ω by finite difference, where the likelihood corresponding to parameter perturbations $\hat{\theta} - \epsilon$ and $\hat{\theta} + \epsilon$ are obtained using the same innovations for distress/belief simulations used for parameter estimation. The unconditional expectation is approximated by a paired bootstrap-simulation procedure: bootstrap to marginalize with respect to dividends exploiting their observability, Monte-Carlo simulation to marginalize with respect to unobservable distress/recovery events and beliefs. We have

$$E[\frac{\partial \log \phi(y_t^i, y_t^j | \omega^i, \hat{\theta})}{\partial \theta}' \frac{\partial \log \phi(y_t^i, y_t^j | \omega^i, \hat{\theta})}{\partial \theta}] \approx \frac{1}{SA} \sum_{u=1}^{SA} \frac{\partial \log \phi(y_{ut}^i, y_{ut}^j | \omega^i, \hat{\theta})}{\partial \theta}' \frac{\partial \log \phi(y_{ut}^i, y_{ut}^j | \omega^i, \hat{\theta})}{\partial \theta} \tag{204}$$

In each of the SA samples, a dividend growth series obtained with a moving block bootstrap (Politis and Romano (1992)) from the original series is combined with a beliefs/events series simulated with the discretization schemes.[26]

Simulation procedure to assess Error-in-Variables biases. Assuming that CAPM betas and exogeneity measures $\overline{\mathcal{E}}_i^{1m.}$ have independent estimation errors, we extract $n = 2000$ independent samples of post-ranking portfolio betas and portfolio dividend-growth parameters from their respective asymptotic distributions:

$$\tilde{\beta}_j^i = \hat{\beta}_j + s_{\beta_j} \epsilon_j^i \quad j = 1, \ldots, 100 \quad i = 1, \ldots, n \tag{205}$$

$$\tilde{\theta}_{jz}^i = \hat{\theta}_{jz} + \mathcal{S}_{\theta_{jz}}' \cdot u_{jz}^i \quad j, z = 1, \ldots, 16 \quad j \neq z \quad i = 1, \ldots, n \tag{206}$$

$\hat{\beta}_j$ is the post-ranking beta estimate for portfolio j, obtained from a time-series regression of its excess returns on market excess returns on the whole sample. Portfolio j is one of the 100 beta-size. s_{β_j} is the asymptotic standard deviation of $\hat{\beta}_j$ and $\epsilon_j^i \sim N(0, 1)$. $\hat{\theta}_{jz}$ is the dividend-growth parameters vector (195) obtained in the pairwise estimation on portfolios j and z. j and z are distinct portfolios among the 16 beta-size sorted on which we have estimated the structural model. $\mathcal{S}_{\theta_{jz}}$ is the factor in the Cholesky decomposition of the inverse of the Fisher information matrix (204), and $u_{jz}^i \sim N(0, 1)$. From $\tilde{\theta}_{jz}^i$, we obtain the $i-$th sampled exogeneity measure as in (18). We then assign betas and exogeneities to the individual stocks and repeat the Fama-McBeth regressions. Iterating for all $n = 2000$ parameters samples, we obtain an empirical distribution for the slopes and for their Fama-McBeth t-statistics implied by measurement errors in variables.

[26]We have used a 5-quarters block size in the moving block bootstrap.

Table I – Post ranking betas

Panel 1

Post ranking betas: market beta-size deciles

	ME avg	β-1	β-2	β-3	β-4	
β avg		0.97528	0.88891	1.02157	1.15979	
ME- 1	1.58197	1.83754	1.08489	1.12965	1.58519	
ME- 2	1.51028	0.90229	1.30433	1.46210	1.32642	
ME- 3	1.34316	0.75287	0.93142	1.22465	1.34157	
ME- 4	1.33726	1.12345	0.77815	1.06159	1.08398	
ME- 5	1.32206	0.88418	0.93034	0.99587	1.25346	
ME- 6	1.28896	0.97540	0.87045	0.99541	1.07308	
ME- 7	1.26103	0.93300	0.78632	0.96284	1.13245	
ME- 8	1.20495	0.79297	0.79869	0.84713	1.00299	
ME- 9	1.13947	0.83599	0.73546	0.81013	0.95171	
ME-10	1.04158	0.71515	0.66905	0.72632	0.84706	
	β-5	β-6	β-7	β-8	β-9	β-10
β avg	1.21445	1.33161	1.40269	1.53384	1.66731	1.83526
ME- 1	1.42728	1.79499	1.65703	1.67928	1.51482	2.10907
ME- 2	1.44504	1.44612	1.59326	1.87417	1.86161	1.88746
ME- 3	1.28562	1.39570	1.47167	1.70486	1.86597	1.45724
ME- 4	1.26397	1.41133	1.44458	1.53519	1.78473	1.88564
ME- 5	1.27834	1.38266	1.42673	1.53899	1.67694	1.85309
ME- 6	1.17294	1.19467	1.41066	1.52175	1.76894	1.90633
ME- 7	1.20313	1.27583	1.26726	1.47736	1.63730	1.93478
ME- 8	1.11154	1.18537	1.30896	1.40619	1.69465	1.90096
ME- 9	1.05148	1.18596	1.25996	1.35501	1.45455	1.75446
ME-10	0.90518	1.04348	1.18678	1.24563	1.41356	1.66360

Panel 2

Post ranking betas: BE/ME deciles

BE/ME-1	BE/ME-2	BE/ME-3	BE/ME-4	BE/ME-5
1.53270	1.46637	1.30274	1.21803	1.16232
BE/ME-6	BE/ME-7	BE/ME-8	BE/ME-9	BE/ME-10
1.12021	1.10391	1.20164	1.26708	1.62674

Table II – Average monthly returns of beta-size sorted portfolios

Panel 1

Average monthly returns, July 1963-June 2008: market beta-size deciles

	ME avg	$\beta-1$	$\beta-2$	$\beta-3$	$\beta-4$
β avg		0.026813	0.017856	0.019688	0.018645
ME- 1	0.056345	0.102945	0.054320	0.060361	0.051114
ME- 2	0.033092	0.040786	0.031486	0.041324	0.027683
ME- 3	0.023833	0.034411	0.018815	0.021345	0.029494
ME- 4	0.018098	0.022097	0.019318	0.016769	0.018219
ME- 5	0.014555	0.019142	0.012753	0.013673	0.014885
ME- 6	0.011201	0.011110	0.009002	0.011347	0.012683
ME- 7	0.010222	0.010447	0.010373	0.010262	0.011142
ME- 8	0.008572	0.011159	0.008286	0.008215	0.008724
ME- 9	0.007957	0.009522	0.008595	0.008515	0.007809
ME- 10	0.004466	0.006507	0.005609	0.005070	0.004700

	$\beta-5$	$\beta-6$	$\beta-7$	$\beta-8$	$\beta-9$	$\beta-10$
β avg	0.018449	0.017880	0.015829	0.016051	0.016296	0.020834
ME- 1	0.050966	0.057902	0.044284	0.045479	0.042596	0.053482
ME- 2	0.038617	0.025836	0.022291	0.030116	0.030117	0.042663
ME- 3	0.018944	0.022313	0.020160	0.019008	0.018385	0.035451
ME- 4	0.018039	0.015908	0.015951	0.014532	0.020035	0.020116
ME- 5	0.020343	0.014244	0.011708	0.012506	0.013020	0.013275
ME- 6	0.009549	0.012666	0.011353	0.009763	0.008461	0.016073
ME- 7	0.008655	0.009947	0.012560	0.010376	0.009352	0.009105
ME- 8	0.007723	0.006859	0.008037	0.008330	0.010601	0.007783
ME- 9	0.007059	0.008841	0.008214	0.007370	0.005851	0.007799
ME- 10	0.004597	0.004283	0.003735	0.003031	0.004536	0.002597

Panel 2

Average monthly returns, July 1963-June 2008: BE/ME deciles

BE/ME-1	BE/ME-2	BE/ME-3	BE/ME-4	BE/ME-5	BE/ME-6	BE/ME-7
0.009868	0.012946	0.009243	0.010183	0.010792	0.012064	0.013736

BE/ME-8	BE/ME-9	BE/ME-10
0.017498	0.021388	0.082819

Table III – 1 month

dynamic centrality measures of beta-size sorted portfolios. Average monthly returns of 5 value-weighted portfolios, formed after sorting stocks according to their (beta-size sorted) portfolio's dynamic centrality. Monthly returns (R_t^{ls}) of a portfolio long the last and short the first quartile of the centrality distribution are regressed (from July 1963 to June 2007) on an intercept, the market excess return, the HML and SMB factors.

Panel 1. 1-month dynamic centrality of beta-size sorted portfolios

	$\beta-1$	$\beta-2$	$\beta-3$	$\beta-4$
ME- 1	3692.75	-29.33	-384.57	12965.45
ME- 2	199.42	1471.52	757.74	3253.19
ME- 3	159.60	244.60	486.73	1583.43
ME- 4	-17.65	-20.42	-7.90	-1812.56

Panel 2. Dynamic centrality sorted portfolios

a) Average monthly returns, July 1963-June 2008

$\overline{\mathcal{DC}}^{\tau}$-1	$\overline{\mathcal{DC}}^{\tau}$-2	$\overline{\mathcal{DC}}^{\tau}$-3	$\overline{\mathcal{DC}}^{\tau}$-4	$\overline{\mathcal{DC}}^{\tau}$-5
0.00483	0.00522	0.0106	0.0116	0.0219

b) Long-short portfolio returns:

$$R_t^{ls} = \alpha + \beta(R_t^m - r_t) + \beta_{hml} HML_t + \beta_{smb} SMB_t + \epsilon_t$$

α	β	β_{hml}	β_{smb}	R^2
0.0126	0.069	0.185	1.421	58.6%
(7.5)	(1.64)	(2.94)	(26.04)	

Table IV

– Time series averages of slopes in Fama-McBeth regressions for monthly returns of stocks. Betas and exogeneity measures are computed after double sorting stocks into beta-size deciles and BM/ME-size deciles. In parenthesis: t-stats, computed with time series standard deviations of coefficients.

Average Slopes of Fama-McBeth regressions, July 1963-June 2008

Factors \ Model	1	2	3	4	5	6
			beta-size sorted portfolios			
β	0.0025 (0.87)	0.00372 (1.317)	0.0087 (3.01)		0.0196 (6.362)	
$\log(ME)$	-0.00454 (-8.21)	-0.00484 (-9.112)				
BE/ME	0.0031 (6.835)	0.0031 (6.835)	0.0031 (6.90)			0.0031 (6.946)
$\overline{\mathcal{DC}}^{1m.}/1000$	0.00054 (2.12)		0.00152 (6.21)	0.0025 (6.96)		
R^2 avg	6.23% (std : 13.40%)	6.09% (std : 13.42%)	5.60% (std : 13.43%)	0.53% (std : 1.03%)	1.05% (std : 2.14%)	4.28% (std : 13.56%)

Table V – Summary statistics of the simulated distribution of (time-series) average slopes and t-statistics of the second stage Fama-McBeth regressions for beta-size sorted portfolios. The cross-sectional model of returns tested is: $R_j = a + b_1\beta_j + b_2(BE/ME)_j + b_3 \log(ME_j) + b_4\overline{\mathcal{DC}}_j^{1m} + \epsilon_j$. Mode slopes and t-statistics are computed with a kernel density estimation, using a Gaussian kernel and the following bandwidths: β slope: 0.00037; $\overline{\mathcal{DC}}^{1m}$ slope: 0.0039; β t-stat: 0.1234; $\overline{\mathcal{DC}}^{1m}$ t-stat: 0.1831.

Monte-Carlo slopes and t-statistics of Fama-McBeth regressions adjusted for error-in-variables, July 1963-June 2008

	β	$\overline{\mathcal{DC}}^{1m}/1000$
avg slope	0.0033	0.00059
median slope	0.0033	0.00063
mode slope	0.0035	0.00065
slope std.	0.0017	0.00017
slope 5%(95%)-*ile*	0.00031 (0.0060)	0.00026 (0.00078)
avg t-stat	1.187	1.869
median t-stat	1.247	1.982
mode t-stat	1.4027	2.072
t-stat std.	0.583	0.578
t-stat 5%(95%)-*ile*	0.1301 (2.053)	0.92 (2.43)

Table VI – Average monthly returns and performance attribution regression of 4 long-short portfolios, each formed of stocks in the same average incoming correlation quartile Θ^j, $j = 1, \ldots, 4$. The portfolio bought (sold) is composed of quartiles of stocks whose most connected trees had the best (worst) returns in the previous month. See Section D.D.1 for details of portfolio formation. The sample period is July 1963-June 2008. T-statistics are in parenthesis. Long-short portfolio returns are regressed in time on an intercept, the market excess return, the HML and SMB factors.

Returns on long-short portfolios formed according to incoming network connectivity, July 1963-June 2008

Θ^j	Avg. return	α	β	β_{smb}	β_{hml}	R^2
y=1	0.01851 (4.622)	0.0176 (4.297)	0.0550 (0.539)	0.3941 (2.9678)	−0.0629 (−0.41)	2.32%
y=2	0.0025 (1.663)	0.0026 (1.730)	−0.0427 (−1.100)	0.072 (1.433)	−0.0378 (−0.65)	0.58%
y=3	0.0033 (1.559)	0.0042 (1.892)	−0.0512 (−0.929)	−0.157 (−2.205)	−0.0525 (−0.63)	1.30%
y=4	0.0092 (2.561)	0.00955 (2.561)	−0.127 (−1.368)	0.0963 (0.796)	0.0101 (−0.072)	0.438%

Table VII – Time series averages of slopes in Fama-McBeth regressions for monthly returns of stocks. Factors $x^{i,cove,size}$, $cove = h, l$, $size = s, m_1, m_2, b$ (as defined in equation (20)) and controls β, BE/ME, $MTCONT.$, REV are computed for stocks in the beta-size sorted portfolios, intersected with analyst coverage information from the I/B/E/S database. In parenthesis: t-stats, computed with time series standard deviations of coefficients.

Average Slopes of Fama-McBeth regressions, July 1983-June 2008

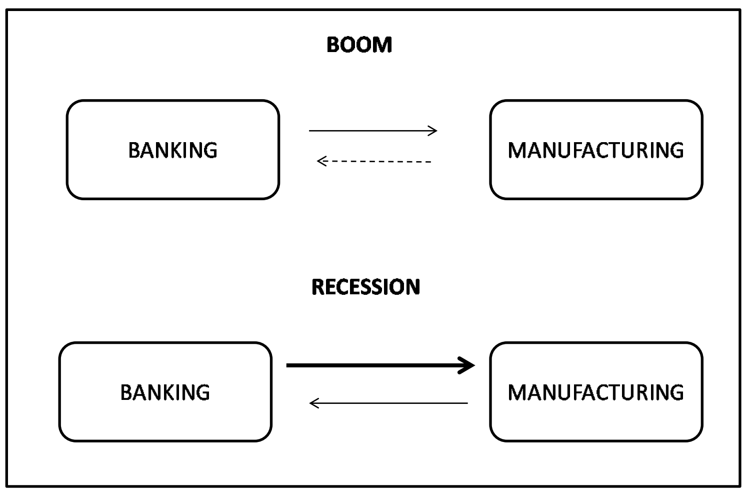
Size	Coverage	Factors	1	2	3
s	l	$x^{i,l,s}$	0.325 (2.186)	0.357 (2.332)	0.501 (2.784)
	h	$x^{i,h,s}$	−23.05 (−0.909)	−22.71 (−0.913)	−23.22 (−0.896)
m_1	l	x^{i,l,m_1}	0.0228 (0.258)	0.0336 (0.398)	0.076 (0.850)
	h	x^{i,h,m_1}	0.056 (0.389)	0.0688 (0.501)	0.1233 (0.897)
m_2	l	x^{i,l,m_2}	−0.069 (−0.787)	−0.0684 (−0.843)	−0.0745 (−0.834)
	h	x^{i,h,m_2}	−0.129 (−1.475)	−0.138 (−1.710)	−0.140 (−1.595)
b	l	$x^{i,l,b}$	−0.102 (−0.880)	−0.124 (−1.196)	−0.207 (−1.814)
	h	$x^{i,h,b}$	−0.165 (−1.405)	−0.195 (−1.839)	−0.270 (−2.366)
		β,BE/ME	yes	yes	no
		REV,$MTCONT.$	yes	no	no
		R^2	4.995% (7.99%)	3.90% (7.01%)	0.87% (1.65%)

Table VIII – Two-trees economy of section II.B. Panel 1 reports the state-contingent intensities of positive and negative endowment jumps of the Manufacturing (M) and the Banking (B) sector. ND means no distress, D means distress. $S = 0$ is the boom state, $S = 1$ is the recession state. Panel 2 gives a schematic representation of the economy. Panel 3 reports a 30-year trajectory of the posterior probability that both sectors will have a distress transition in 1 year. When M and B are not connected, intensities assume in all states the value they have in (ND, ND).

Panel 1

(B,M)	(ND,ND)		(D,ND)		(ND,D)		(D,D)	
	$S=0$	$S=1$	$S=0$	$S=1$	$S=0$	$S=1$	$S=0$	$S=1$
λ^M	0.2	0.4	0.6	1.8	-		-	
λ^B	0.1	0.16	-		0.2	0.6	-	
η^M	-		-		0.4	0.2	0.4	0.2
η^B	-		0.4	0.2	-		0.4	0.4

Panel 2



Panel 3

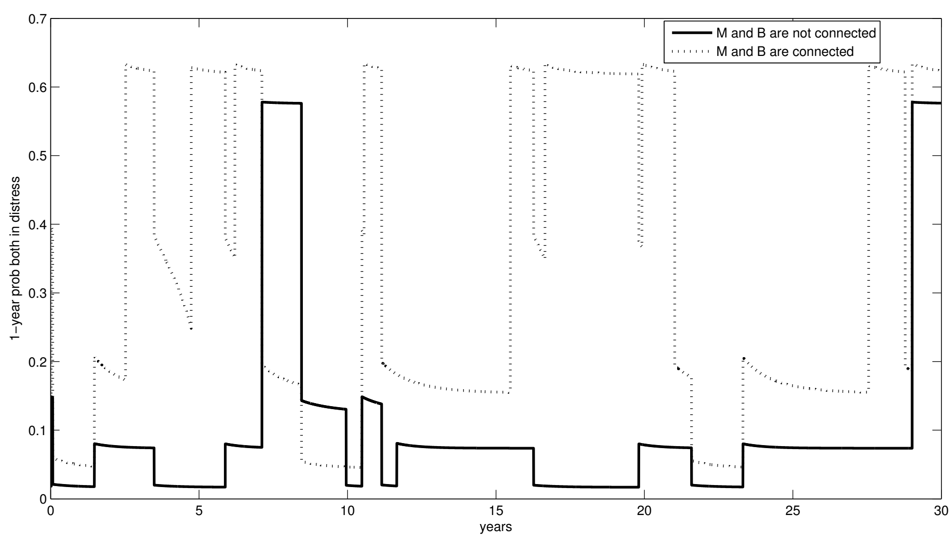


Table IX – Panel
1 reports expected returns and exogeneity measures obtained from returns, dividends and beliefs simulated from the model, for the 10 trees economy described in Section V.A. It also reports the average returns and t-statistics of a portfolio long the sectors with largest dynamic centrality and short those with the smallest. Panel 2 plots expected returns-dynamic centrality pairs for each Sector, as well as a fitted linear model.

Panel 1

Sector	Expected Return	$\overline{\mathcal{DC}}_i^{1\,month}$
1- Agriculture	0.1351	22.509
2- Mining	0.1280	11.324
3- Utilities	0.1359	11.987
4- Construction	0.1002	4.278
5- Manufacturing	0.0631	-2.154
6- Wholesail Trade	0.0805	9.256
7- Retail	0.1359	5.862
8- Transportation	0.1408	8.476
9- Information	0.1394	7.163
10- Others	0.1201	-78.703
Exogeneity Test		
Long-Short (buy 1,3 - sell 4,5)	0.0877	
(t-stat)	(14.86)	

Panel 2

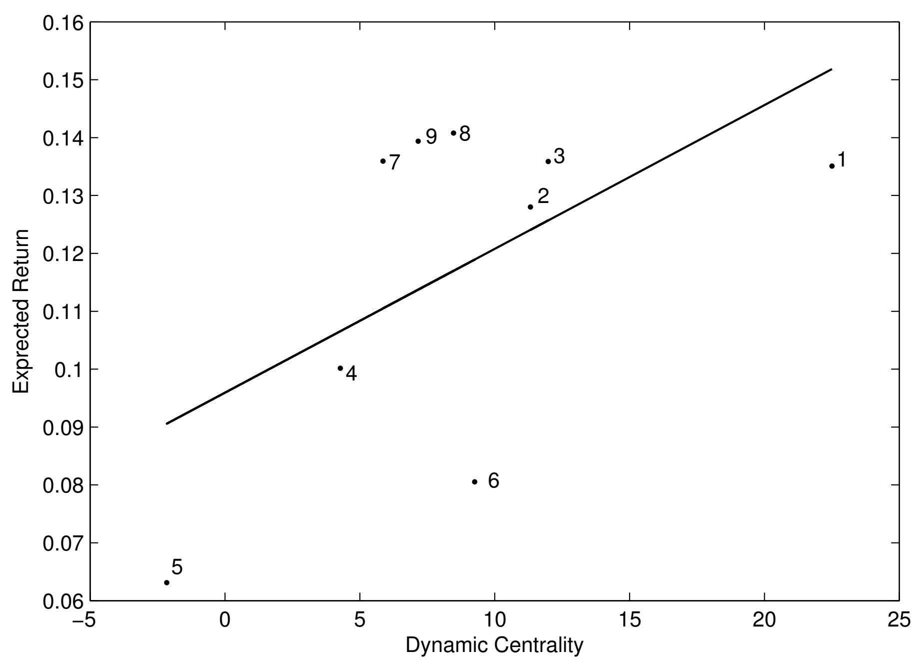


Table X – For the 'Star' network of Proposition 4, the table reports the critical k^* for different parameter combinations. It also reports the percentage of states for which condition $p_t^{t_u}(H^1) < p_t^{t_u}(\overline{H^1})$ is violated when $k^i < k^*$, where k^i is 1.05 for $i = 1$, k^* for $i = 6$, and is linearly increasing in i. $t_u - t = 1$ year.

(λ, η)	k^*	% of violations				
		k^1	k^2	k^3	k^4	k^5
(0.40 , 0.50)	22.02	61.72	22.05	6.58	1.29	0.28
(0.43 , 0.50)	19.96	57.77	19.55	5.41	1.29	0.28
(0.47 , 0.50)	18.23	54.30	17.17	5.03	1.08	0.27
(0.50 , 0.50)	16.74	49.52	14.98	4.48	1.08	0.27
(0.50 , 0.40)	15.72	45.34	14.17	4.48	1.14	0.27
(0.50 , 0.43)	16.15	47.42	14.42	4.48	1.02	0.27
(0.50 , 0.47)	16.61	49.41	15.39	4.47	1.08	0.27
(0.50 , 0.50)	17.13	50.75	15.41	4.48	1.08	0.27

Table XI – Representation of exogeneity estimates on beta-size sorted portfolios. Portfolios' exogeneity are obtained as in expression (18). Smaller circles means lower exogeneity. The network connectivity is reported as directed arrows only for connections with strength beyond a given threshold ($\mathcal{E}_{ij}^{1m.} > 0$).